Solve the following problem using the Euler method. Recall this method from your text:

\[ y' = f(x, y) \]

by defining a small \( \Delta x \), called \( h \). Then

\[ y_{n+1} = y_n + h \cdot f(x_n, y_n) \]

with initial condition

\[ y_0 = y(0) \]

Solve \( y' = y + x^2 \) where \( y' = f(x, y) \)
Subject to \( y(0) = 1 \).

1) Select a cell for the value (\( h = 0.2 \)) of \( h \).
   in cell B2 type \( h = \)
   in cell C2 type \( 0.2 \)
You might want to format cell B2 to bold and right-justified and C2 to left-justified.

2) Put column titles in cells A4, B4 and C4
   in cell A4 type \( x_n \)
   in cell B4 type \( y_n \)
   in cell C4 type \( f(x_n, y_n) \)
You might want to format these cells to bold and center-justified.

3) Enter the initial conditions
   in cell A5 type \( 0 \)
   in cell B5 type \( 1 \)
   in cell C5 type \( =B5+A5^2 \)
Note that you have entered a formula in cell C5. In the formula you refer to the values found in two other cells, B5 and A5.

4) Enter the first line of the Euler iteration in row 6
   in cell A6 type \( =A5+$C$2 \) the new value of \( x \)
   in cell B6 type \( =B5+$C$2*C5 \) the new value of \( y \)
   \( \text{ynew} = \text{yold} + h \cdot f(xold, yold) \)
The use of the $ sign is absolute referencing. In the next step we will copy formulas from row 6 down. The cell references that have no $ sign will be incremented while the cell $C$2 will remain.

5) Select the three cells A6, B6 and C6 simultaneously by clicking in cell A6 then with the left mouse button down, dragging over to C6 so all three cells are selected. Release the left mouse button.
Note that there is a small square in the lower right-hand corner of the selection rectangle. Move the cursor (mouse) over this corner until the cursor changes. Left mouse click then drag down to include A15, B15 and C15. Let go of the left mouse button. Your screen should look like the following (c.f. Table 1.3).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>h=</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>xn</td>
<td>yn</td>
<td>f(xn,yn)</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>1.2</td>
<td>1.24</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>1.448</td>
<td>1.608</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>1.7696</td>
<td>2.1296</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>2.19552</td>
<td>2.83552</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>2.762624</td>
<td>3.762624</td>
</tr>
<tr>
<td>11</td>
<td>1.2</td>
<td>3.515149</td>
<td>4.955149</td>
</tr>
<tr>
<td>12</td>
<td>1.4</td>
<td>4.506179</td>
<td>6.466179</td>
</tr>
<tr>
<td>13</td>
<td>1.6</td>
<td>5.799414</td>
<td>8.359414</td>
</tr>
<tr>
<td>14</td>
<td>1.8</td>
<td>7.471297</td>
<td>10.7113</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>9.613557</td>
<td>13.61356</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Row 7 formulas should be
- in cell A7: =A6+$C$2 the new value of \( x \)
- in cell B7: =B6+$C$2*C6 the new value of \( y \)
- in cell C7: =B7+A7^2 the new value of \( f(x,y) \)

6) Now add a column with the correct answer to see how well Euler's method is working.

The actual \( y \) is
\[
y = - x^2 - 2x - 2 + c e^x
\]
where \( c \) is a constant determined by the initial conditions. Since we have chosen \( y(0)=1 \), \( c=3 \) and
- in cell D5: =3*EXP(A5)-A5^2-2*A5-2

The spreadsheet now looks like the following:
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>h=</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(x_n)</td>
<td>(y_n)</td>
<td>(f(x_n,y_n))</td>
<td>(y(x))</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>1.2</td>
<td>1.24</td>
<td>1.224208</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>1.448</td>
<td>1.608</td>
<td>1.515474</td>
</tr>
<tr>
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<td>1.7696</td>
<td>2.1296</td>
<td>1.906356</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>2.19552</td>
<td>2.83552</td>
<td>2.436623</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>2.762624</td>
<td>3.762624</td>
<td>3.154845</td>
</tr>
<tr>
<td>11</td>
<td>1.2</td>
<td>3.515149</td>
<td>4.955149</td>
<td>4.120351</td>
</tr>
<tr>
<td>12</td>
<td>1.4</td>
<td>4.506179</td>
<td>6.466179</td>
<td>5.4056</td>
</tr>
<tr>
<td>13</td>
<td>1.6</td>
<td>5.799414</td>
<td>8.359414</td>
<td>7.099097</td>
</tr>
<tr>
<td>14</td>
<td>1.8</td>
<td>7.471297</td>
<td>10.7113</td>
<td>9.308942</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>9.613557</td>
<td>13.61356</td>
<td>12.16717</td>
</tr>
</tbody>
</table>

Note that these numbers are the long version of those presented in section 1.4 of your text.

7) In order to determine how accurate the solution is, plot the numerical and analytical solutions.
   Select A4→B15 by selecting A4 with the left mouse button, then dragging down to B15.
   Press the CTRL button then, holding it down, with the left mouse button select D4→D15.
Columns A, B, and D rows 4→15 should be selected, i.e. titles and data.
Press the Chart Wizard button, then answer the questions.

Make sure you select XY (SCATTER) for the plot type, not LINE. Line plots assume the first column consists of titles, not values. In order to space the x-axis values correctly, you must use Scatter plots. You have the option of showing lines only on your plot.

Your first row and column are Xdata and Legend text. Note that your screen shots will be somewhat different as I am using EXCEL 7 for Office 95.
Now enter the plot title and axis titles.

Finally you can reformat parts of your chart by double-clicking the chart, then selecting different parts of it. Experiment to determine what you like.
Note that I have used a solid line for the analytical data and markers for those points that were calculated. I double clicked axis numbers to format the axis and double clicked the gray background to get rid of the shading.

You can see that as $x$ increases, the error in the numerical solution increases. You can either use a smaller step size, $h$, or you can use another method (later).