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# CRACK DEVELOPMENT IN PLANE UNDER THE ACTION OF COMPRESSIVE LOAD

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UDC 539.375

## 1. Formulation of Problem

The initial development of a single crack with a compressive stress acting at some angle $\gamma \in (0; \pi/2)$ to the plane of this crack is considered [5, 6], in the case where approximately rectilinear side arms [1, 2, 7, 8] of length $\ell \ll c$ have begun to appear under the action of the external load at the initial crack AB (Fig. 1). The approximate model of this process is based on the plane elasticity-theory problem [8] with boundary conditions

$$\tau_{xy} = -\tau_c + \mu\sigma_y; \qquad [v] = [\sigma_y] = 0; \qquad (|x| < c;\ y = 0); \tag{1.1}$$

$$\sigma_{\varphi\varphi} = \tau_{r\varphi} = 0; \qquad (0 \leqslant r \leqslant l,\ \varphi = 0); \tag{1.2}$$

$$\sigma_1(\infty) = \sigma_1; \qquad \sigma_2(\infty) = \sigma_2. \tag{1.3}$$

Here $\mu$ is the frictional coefficient between the sides of the basic crack; $(r, \phi)$ are local polar coordinates in the vicinity of the points $(\pm c;\ 0)$; $\sigma_1$, $\sigma_2$ are the principal stresses at infinity.

Solving the problem in Eqs. (1.1)-(1.3), the stress-intensity coefficients at the tips of the side arms are found. According to one of the known criteria [4], crack development occurs in the direction determined by the condition $K_{II}(\theta, \ell) = 0$. Solving this equation with fixed $\ell$, the angle $\theta = \theta(\ell)$ is found. The length is now assumed to be variable, within the interval $(0, \ell_0)$, $\ell_0 \ll c$. The quantity $\theta_* = \theta(+0)$, as soon as its exists, specifies the angle between the basic crack and the side arms. Suppose that $\theta_*$ is the angle of crack advance. The dependence of this angle on the parameters $\gamma$ and $\mu$ is of interest. Then the local stability of such a crack is determined by the sign of the derivative $\partial K_I(\theta, \ell)/\partial \ell$.

In this approach, the problem reduces to finding the asymptotes of the stress-intensity coefficients for Eqs. (1.1)-(1.3) as $\ell/c \to +0$, the other parameters being fixed. Regardless of the possible interpretations, the significant terms of this asymptote may be taken to be approximate formulas for the given coefficients when $\ell \ll c$, which are useful specifically in those cases where methods of numerical calculation become unreliable.

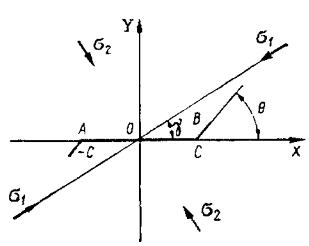


Fig. 1

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 0038-5298/90/2609-0878$12.50 

## 2. Method of Solution

The stress-intensity coefficients at the tip of the side arms are determined by the expression

$$K_{\rm I} + iK_{\rm II} = (2\pi)^{3/2} e^{i\theta} \lim_{r\to l-0} \overline{\alpha(r)} \cdot \sqrt{l-r}, \tag{2.1}$$

where α(r) is the density of the dislocation distribution

$$\alpha(r) = \frac{G \cdot e^{i\theta}}{\pi i(\varkappa+1)}([u_r'] + i[u_\theta']); \qquad (0 \leqslant r \leqslant l).$$

Here G is the shear modulus; κ is the Muskhelishvili constant; the square brackets denote a discontinuity in the quantity which they enclose. The function α(r) is a solution of the singular equation [8]

$$\int_0^l \frac{2e^{i\theta}\overline{\alpha(r)}}{s-r}\,dr + \int_0^l (\alpha(r)\cdot L_1(r,s,\theta) + \overline{\alpha(r)}\cdot L_2(r,s,\theta))\,\mathrm{d}r = Q(s,\theta); \quad 0 \leqslant s \leqslant l. \tag{2.2}$$

Here

$$\begin{aligned} L_1(r,s,\theta) = {} & (1-2e^{2i\theta})[\bar\beta(\bar z_0 - z_0)G(z,z_0) - \beta(F(z,z_0) + F(z,\bar z_0))] + \\ & + \beta(\overline{F(z,z_0)} + \overline{F(z,\bar z_0)}) - \bar\beta(\bar z_0 - z_0)\overline{G(z,\bar z_0)} + e^{2i\theta}(\bar z - z) \times \\ & \times (\bar\beta(\bar z_0 - z_0)G'(z,z_0) - \beta(F'(z,z_0) + F'(z,\bar z_0))) + \frac{\bar z + \bar z_0}{(z+z_0)^2}e^{2i\theta} - \frac{1}{z+z_0}; \end{aligned} \tag{2.3}$$

$$\begin{aligned} L_2(r,s,\theta) = {} & (1-2e^{2i\theta})(\bar\beta(F(z,z_0) + F(z,\bar z_0)) + \beta(\bar z_0 - z_0)G(z,\bar z_0)) - \\ & - \bar\beta(\overline{F(z,z_0)} + \overline{F(z,\bar z_0)}) - \beta(\bar z_0 - z_0)\overline{G(\bar z,\bar z_0)} + e^{2i\theta}(\bar z - z) \times \\ & \times (\bar\beta(F'(z,z_0) + F'(z,\bar z_0)) + (\bar z_0 - z_0)\beta G'(z,\bar z_0)) - \left(\frac{1}{z+z_0} + \frac{e^{2i\theta}}{z+z_0}\right). \end{aligned} \tag{2.4}$$

Also

$$F(z,z_0) = \left(1 - \frac{z}{z_0}\sqrt{\frac{z_0^2-c^2}{z^2-c^2}}\right)\frac{z_0}{z^2-z_0^2}; \tag{2.5}$$

$$\begin{aligned} G(z,z_0) = {} & \left(1 - \frac{z}{z_0}\sqrt{\frac{z_0^2-c^2}{z^2-c^2}}\right)\frac{2z_0^2}{(z^2-z_0^2)^2} + \\ & + \left(1 - \frac{zz_0}{\sqrt{(z^2-c^2)(z_0-c^2)}}\right)\frac{1}{z^2-z_0^2}; \end{aligned} \tag{2.6}$$

$$z = c + se^{i\theta}; \qquad z_0 = c + re^{i\theta}; \qquad \beta = 1/2 + 1/2i\mu.$$

The expression for the right-hand side of Eq. (2.2) takes the form

$$\begin{aligned} Q(s,\theta) = {} & \frac{1}{2}i(\tau_{xy}^\infty - \mu\sigma_y^\infty + \tau_c)\left[\frac{\bar z}{\sqrt{\bar z^2 - c^2}} - \frac{z}{\sqrt{z^2-c^2}} + \right. \\ & \left. + e^{2i\theta}\left(\frac{(\bar z - z)c^2}{(z^2-c^2)^{3/2}} + \frac{2z}{\sqrt{z^2-c^2}} - 2\right)\right] + \frac{\sigma_y^\infty + \sigma_x^\infty}{2} + e^{2i\theta}\left(\frac{\sigma_y^\infty - \sigma_x^\infty}{2} + i\tau_{xy}^\infty\right). \end{aligned} \tag{2.7}$$

Here $\sigma_x^\infty$, $\sigma_y^\infty$, $\tau_{xy}^\infty$ are expressed in terms of the principal stresses $\sigma_1$, $\sigma_2$ by the well-known formulas of plane elasticity theory.

It is simple to verify that, at the point r = s, i.e., at $z_0 = z$, the functions $L_1$ and $L_2$ have eliminable discontinuities at each θ ∈ (0, π/2).

In [8], the results of numerical solution of Eq. (2.2) with subsequent calculation of $K_I$ and $K_{II}$ according to Eq. (2.1) are shown (in graphical form) for a set of parameters ℓ, $\tau_c$,

$\mu$, $\gamma$. As is evident from [8], it is impossible to obtain numerical information on the limiting values of $K_I$ and $K_{II}$ as $\ell \to +0$ even for the parameters employed. The approximate formula proposed in [8]

$$\alpha(r) = \frac{1}{\sqrt{r(l-r)}}\left(a(0)\left(\frac{2r}{l}-1\right) + ib(0)\frac{r}{l}\right) \quad (0 < r < l) \tag{2.8}$$

is also unsuitable in the case where $\ell \ll c$. In fact, according to Eq. (2.8), $\alpha(+0) = \infty$; it is shown that there is no singularity $1/\sqrt{r}$ in the function $\alpha(r)$. In connection with this, the asymptote of the stress-intensity coefficients $K_I$ and $K_{II}$ is obtained directly from Eq. (2.2).

## 3. Asymptotes of Terms of Integral Equation

Considering Eq. (2.2) as an identity when $0 < s < \ell$, pass to the limit as $s \to \ell - 0$. Introducing the notation

$$A(r) = \alpha(r)\cdot\sqrt{l-r}; \tag{3.1}$$

$$I_0(s, \theta) = \int_0^l \frac{2e^{i\theta}\bar{\alpha}(r)}{s-r}\,dr, \tag{3.2}$$

and using elementary methods of asymptotic estimation of given integrals [3], the following relation is obtained for all $\ell > 0$ and $0 < \theta < \pi/2$

$$I_0(l, \theta) + I_1(l, \theta) = Q(l, \theta); \qquad l > 0; \qquad 0 < \theta < \frac{\pi}{2}, \tag{3.3}$$

where

$$I_0(l, \theta) = -2e^{i\theta}\left[\frac{2\overline{A(l)}}{\sqrt{l}} + \int_0^l \frac{\overline{A(l)} - \overline{A(r)}}{(l-r)^{3/2}}\,dr\right]; \tag{3.4}$$

$$I_1(l, \theta) = \int_0^l \left(A(r)\cdot L_1(r, l, \theta) + \overline{A(r)}\cdot L_2(r, l, \theta)\right)\frac{dr}{\sqrt{l-r}}. \tag{3.5}$$

The functions Q, $L_1$, $L_2$ are defined by Eqs. (2.3)-(2.7). It follows from Eq. (2.5) that

$$Q(l, \theta) = \frac{C_0(\theta, \gamma)}{\sqrt{l}} + C_1(\theta, \gamma) + o(1), \quad (l \to +0), \tag{3.6}$$

where

$$C_0(\theta, \gamma) = \frac{i\sqrt{c}}{2\sqrt{2}}(3e^{i\theta} - 1)\cos\frac{\theta}{2}\left((\sigma_1 - \sigma_2)\sin\gamma(\cos\gamma - \mu\sin\gamma) - \mu\sigma_2 + \tau_c\right); \tag{3.7}$$

$$C_1(\theta, \gamma) = (\sigma_1 + \sigma_2)e^{2i\theta}(\beta - ie^{-i\theta}\sin\theta) - (\sigma_1 - \sigma_2)e^{2i\theta}\beta\cos 2\gamma - i\tau_c e^{2i\theta}. \tag{3.8}$$

Thus, Eq. (3.3) leads to the asymptotic equation

$$-2e^{i\theta}\left(\frac{2\overline{A(l)}}{\sqrt{l}} + \frac{1}{\sqrt{l}}\int_0^1 \frac{\overline{A(l)} - \overline{A(\tau l)}}{(1-\tau)^{3/2}}\,d\tau\right) + \sqrt{l}\int_0^1 (A(l\tau)\cdot L_1(l\tau, l, \theta) +$$
$$+ \overline{A(l\tau)}\cdot L_2(l\tau, l, \theta))\frac{d\tau}{\sqrt{1-\tau}} = \frac{C_0(\theta, \gamma)}{\sqrt{l}} + C_1(\theta, \gamma) + o(1). \tag{3.9}$$

Asymptotic expressions for $L_1$ and $L_2$ are obtained from Eqs. (2.3) and (2.4)

$$L_1(l\tau, l, \theta) = \frac{1}{l}L_{10}(\tau, \theta) + O(1), \quad (l \to +0),$$
$$L_2(l\tau, l, \theta) = \frac{1}{l}L_{20}(\tau, \theta) + O(1), \quad (l \to +0) \tag{3.10}$$

and $L_{10}$ and $L_{20}$ are determined using relations which follow from Eqs. (2.3) and (2.4)

$$
\begin{aligned}
&F(z, z_0) = \frac{1}{2l}\frac{e^{-i\theta}}{1+\sqrt{\tau}} + O(1) \quad (l \to +0);\\
&F(z, \bar{z}_0) = \frac{1}{2l}\frac{e^{-i\theta}}{1+\sqrt{\tau}e^{-i\theta}} + O(1) \quad (l \to +0);\\
&(z_0-\bar{z}_0)\,G(z, \bar{z}_0) = -\frac{1}{2l}\frac{ie^{-i\theta}\sin\theta\cdot\sqrt{\tau}}{(1+\sqrt{\tau}e^{-i\theta})^2} + O(1) \quad (l \to +0);\\
&(z_0-\bar{z}_0)\,G(z, z_0) = -\frac{1}{2l}\frac{ie^{-2i\theta}\sin\theta\sqrt{\tau}}{(1+\sqrt{\tau})^2} + O(1) \quad (l \to +0);\\
&(z-\bar{z})\,F'_z(z, z_0) = -\frac{1}{2l}\frac{ie^{-2i\theta}\sin\theta\,(2+\sqrt{\tau})}{(1+\sqrt{\tau})^2} + O(1) \quad (l \to +0);\\
&(z-\bar{z})\,F'_z(z, \bar{z}_0) = -\frac{1}{2l}\frac{ie^{-2i\theta}\sin\theta\,(2+\sqrt{\tau}e^{-i\theta})}{(1+\sqrt{\tau}e^{-i\theta})^2} + O(1) \quad (l \to +0);\\
&(\bar{z}-z)(z_0-\bar{z}_0)\,G'_z(z, \bar{z}_0) = \frac{1}{2l}\frac{e^{-2i\theta}\sin^2\theta\,(3+\sqrt{\tau}e^{-i\theta})\sqrt{\tau}}{(1+\sqrt{\tau}e^{-i\theta})^3} + O(1) \quad (l \to +0);\\
&(\bar{z}-z)(z_0-\bar{z}_0)\,G'_z(z, z_0) = \frac{1}{2l}\frac{e^{-3i\theta}\sin^2\theta\,(3+\sqrt{\tau})\sqrt{\tau}}{(1+\sqrt{\tau})^3} + O(1) \quad (l \to +0);\\
&\frac{z+\bar{z}_0}{(z+z_0)^2}e^{2i\theta} - \frac{1}{(z+z_0)} = O(1) \quad (l \to +0);\\
&\frac{1}{z+\bar{z}_0} + \frac{e^{2i\theta}}{z+z_0} = O(1) \quad (l \to +0); \quad \left(z = c + se^{i\theta},\ z_0 = c + re^{i\theta},\ \theta \in \left]0, \frac{\pi}{2}\right[\right).
\end{aligned}
\tag{3.11}
$$

## 4. Expansion of A(r) in Generalized Power Series

Series expansion of A(r) in Eq. (3.1) in powers of $\sqrt{(\ell - r)}$ $(0 < r < \ell)$ and substitution of the result in Eq. (3.9) leads to the conclusion that the singularity of A(r) at the point r = +0, where it exists, takes the form

$$A(r) \sim a(0)\cdot r^{-k} \quad (r \to +0). \tag{4.1}$$

In connection with this, A(r) is sought in the form

$$A(r) = r^{\delta-1}(a_0(\theta,\gamma) + a_1(\theta,\gamma)\cdot r + \ldots) + (b_0(\theta,\gamma) + b_1(\theta,\gamma)\cdot r + \ldots). \tag{4.2}$$

Let T denote the additive integral operator on the left-hand side of Eq. (3.9). Then

$$
\begin{aligned}
T[ar^{\delta-1}] = {} & (aP_1^1(\delta,\theta) + \bar{a}P_1^2(\delta,\theta))\, l^{\delta-3/2}(aP_2^1(\delta,\theta) + \bar{a}P_2^2(\delta,\theta))\, l^{\delta-1/2} + \\
& + o(1)\,(l \to +0),
\end{aligned}
\tag{4.3}
$$

where $P_1^1$, $P_1^2$, $P_2^1$, $P_2^2$ do not depend on $\ell$. Hence and from Eq. (4.2), it follows that

$$
\begin{aligned}
T[A(r)] = {} & (a_0(\theta,\gamma)\cdot P_1^1(\delta,\theta) + \overline{a_0(\theta,\gamma)}\cdot P_1^2(\delta,\theta))\, l^{\delta-3/2} + (b_0(\theta,\gamma)\cdot P_1^1(1,\theta) + \\
& + \overline{b_0(\theta,\gamma)}\cdot P_1^2(1,\theta))\, l^{-1/2} + (a_0(\theta,\gamma)P_2^1(\delta,\theta) + \overline{a_0(\theta,\gamma)}\cdot P_2^2(\delta,\theta) + \\
& + a_1(\theta,\gamma)\cdot P_1^1(\delta+1,\theta) + \overline{a_1(\theta,\gamma)}\,P_1^2(\delta+1,\theta))\, l^{\delta-1/2} = \frac{C_0(\theta,\gamma)}{\sqrt{l}} + C_1(\theta,\gamma) + o(1) \quad (l \to +0) \quad (0 < \delta \leqslant 1).
\end{aligned}
\tag{4.4}
$$

It may be concluded on the basis of Eq. (4.4) that $\delta$ may take the value 1/2, so that Eq. (4.2) takes the form

$$A(r) = \frac{a_0(\theta,\gamma)}{\sqrt{r}} + b_0(\theta,\gamma) + a_1(\theta,\gamma)\cdot\sqrt{r} + \ldots \quad (0 < r < l). \tag{4.5}$$

Under the condition in Eq. (4.5), Eq. (4.4) leads to the relation

$$a_0(\theta,\gamma)\cdot P_1^1(1/2,\theta) + \overline{a_0(\theta,\gamma)}\,P_1^2(1/2,\theta) = 0 \quad \theta \in \left]0; \frac{\pi}{2}\right[. \tag{4.6}$$

Equation (4.6) is a linear homogeneous system in terms of Re $a_0$ and Im $a_0$, with the determinant

$$\Delta(\theta) = \left|P_1^1\left(\frac{1}{2}, \theta\right)\right|^2 - \left|P_1^2\left(\frac{1}{2}, \theta\right)\right|^2.$$

Values of $\Delta(\theta)$ are found for $\theta = 1°\cdot K$, K = 1, 2, ..., 89 and $\mu$ = 0, 1·m; m = 0, 1, ..., 8, with a guaranteed absolute error of $10^{-3}$. It is found that, with any of the given combinations of k and m, the corresponding values of the determinant lie on smooth curves and these values are known to be nonzero. Thus, it remains that $a_0(\theta, \gamma) = 0$.

Note. It follows from the foregoing that, with all the given $\theta$, the function $\alpha(r) = A(r)/\sqrt{(\ell - r)}$ does not have an integrable singularity at r = +0, according to Eq. (4.5). Without constituting strict proof of this result, the given calculations show that the opposite is unlikely to be true. Finally, in view of Eqs. (3.9), (3.10), and (4.5), the asymptotic equality for A(r) takes the form

$$-2e^{i\theta}\left(\frac{2\overline{A(l)}}{\sqrt{l}} + \frac{1}{\sqrt{l}}\int_0^1 \frac{\overline{A(l)} - \overline{A(l\tau)}}{(1-\tau)^{3/2}}\,d\tau + \frac{1}{\sqrt{l}}\int_0^1 (A(l\tau)\cdot L_{10}(\tau, \theta) + \right.$$

$$\left. + \overline{A(l\tau)}\cdot L_{20}(\tau, \theta))\frac{d\tau}{\sqrt{1-\tau}}\right) = \frac{C_0(\theta, \gamma)}{\sqrt{l}} + C_1(\theta, \gamma) + o(1) \quad (l \to +0).$$

This determines the asymptote of A(r) and also, according to Eqs. (2.1) and (3.1), the asymptote of the stress-intensity coefficients.

## 5. Asymptotes of Stress-Intensity Coefficients. Some Conclusions

Assuming that $a_0(\theta, \gamma) = 0$ in Eq. (4.5) and reverting to Eq. (2.1), it is found that

$$(2\pi)^{-3/2}e^{-i\theta}(K_I + iK_{II}) = A(l, \theta, \gamma) = b_0(\theta, \gamma) + a_1(\theta, \gamma)\sqrt{l} + o(\sqrt{l}), \quad (l \to +0), \tag{5.1}$$

where $a_1(\theta, \gamma)$ and $b_0(\theta, \gamma)$ are determined from the system of equations

$$b_0(\theta, \gamma)P_1^1(1, \theta) + \overline{b_0(\theta, \gamma)}\cdot P_1^2(1, \theta) = C_0(\theta, \gamma); \quad a_1(\theta, \gamma)P_1^1(3/2, \theta) + \overline{a_1(\theta, \gamma)}\cdot P_1^2(3/2, \theta) = C_1(\theta, \gamma), \tag{5.2}$$

the right-hand sides $C_0(\theta, \gamma)$ and $C_1(\theta, \gamma)$ of which are determined by Eqs. (3.7) and (3.8), while the coefficients P are obtained from Eqs. (3.10), (4.4), and (4.7) with $\delta = 1/2$, $a_0(\theta, \gamma) = 0$. The expression for $b_0(\theta, \gamma)$ takes the form

$$b_0(\theta, \gamma) = \frac{1}{|P_1^1(1, \theta)|^2 - |P_1^2(1, \theta)|^2}\left(\mathrm{Re}\,(P_1^1(1, \theta) - P_1^2(1, \theta))\overline{C_0(\theta, \gamma)} - \mathrm{Im}\,(P_1^1 + P_1^2)\,\overline{C_0(\theta, \gamma)}\right). \tag{5.3}$$

The functions $P_1^1(1, \theta)$ and $P_1^2(1, \theta)$ are expressed in terms of the function $\Phi(z)$ and its first two derivatives, where

$$\Phi(z) = \int_0^1 \frac{dx}{\sqrt{x}\sqrt{1-x}(1+z\sqrt{x})} = \frac{4}{z+1}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{z-1}{z+1}\right)^n;$$
$$z = e^{i\theta}, \quad \theta \in (0, \pi/2). \tag{5.4}$$

As is evident from Eq. (5.4), $\Phi(z)$, $\Phi'(z)$, $\Phi''(z)$ are series of terms with alternating signs in powers of $\tan(\theta/2)$, which simplifies their calculation and the estimation of the error. In the second relation in Eq. (5.2), $P_1^1(3/2, \theta)$ and $P_1^2(3/2, \theta)$ are expressed in the same manner in terms of $\Phi(z)$ and its derivatives. Note also that according to Eqs. (3.7) and (5.3)

$$b_0(\theta, \gamma) = ((\sigma_1 - \sigma_2)\sin\gamma(\cos\gamma - \mu\cdot\sin\gamma) - \mu\sigma_2 + \tau_c)\,b(\theta). \tag{5.5}$$

From Eqs. (5.3)-(5.5) and the analogous formulas for $a_1(\theta, \gamma)$, the stress-intensity coefficients $K_I(\theta, \gamma, \mu)$ and $K_{II}(\theta, \gamma, \mu)$ are calculated using Eq. (5.1).

The results of these calculations permit the following conclusions.

The coefficient $\mathrm{Re}(a_1(\theta)e^{i\theta})$ of the asymptotic Eq. (5.1) is negative with any choice of parameters. Accordingly, crack growth is stable in the initial stage of its development.

At small angles $\gamma$, as follows from Eqs. (3.7) and (5.5)

$$\begin{gathered} b_0(\theta,\gamma) = (-\mu\sigma_2 + \tau_c + (\sigma_1 - \sigma_2)\gamma)\, b(\theta) + O(\gamma^2), \quad (\gamma \to +0); \\ a_1(\theta,\gamma) = a_1(\theta) + O(\gamma^2) \quad (\gamma \to +0), \end{gathered} \tag{5.6}$$

therefore, the expression for the stress-intensity coefficients at the end of the side arms at small $\gamma$ takes the form

$$K_I + iK_{II} = (2\pi)^{3/2}(-\mu\sigma_2 + \tau_c + (\sigma_1 - \sigma_2)\gamma)\, b(0)\, e^{i0} + a_1(\theta)\, e^{i\theta}\sqrt{l} + O(l + \gamma^2), \quad (l \to +0). \tag{5.7}$$

If the expression in parentheses in Eq. (5.7) is zero, the stress-intensity coefficients tend to zero with decrease in length $\ell$. In particular, when $\gamma = 0$ and $\tau_c = \sigma_2 = 0$, $K_I + iK_{II} = O(\sqrt{\ell})$ $(\ell \to +0)$.

The stress-intensity coefficients calculated in the present work with $\ell/c \cong 10^{-2}$-$10^{-1}$ are close to the corresponding numerical results [8], although they are obtained by a fundamentally different method.

The limiting values of

$$k_1 = \frac{K_I}{\sqrt{\pi c}\,((\sigma_1 - \sigma_2)\sin\gamma\,(\cos\gamma - \mu\sin\gamma) - \mu\sigma_2 + \tau_c)};$$

$$k_2 = \frac{K_{II}}{\sqrt{\pi c}\,((\sigma_1 - \sigma_2)\sin\gamma\,(\cos\gamma - \mu\sin\gamma) - \mu\sigma_2 + \tau_c)}$$

do not depend on the angle of application of the external load $\gamma$. The dependence of these quantities on the break angle of the crack when $\tau_c$ is shown in Figs. 2 and 3. As is evident from Figs. 2 and 3, $K_1$ decreases with increase in firctional coefficient.

As is evident from Figs. 2 and 3, $K_I(\theta)$ reaches a maximum as $\ell/c \to 0$ with fixed $\gamma$ and $\mu$ and $K_{II}(\theta)$ vanishes at the same value of $\theta_* = \theta_*(\mu)$. Hence, $K_I^2 + K_{II}^2$ also reaches a maximum at the same value of $\theta_*$, i.e., the use of any of the widespread criteria of crack growth in the given problem [4] gives the same value of $\theta_*$.

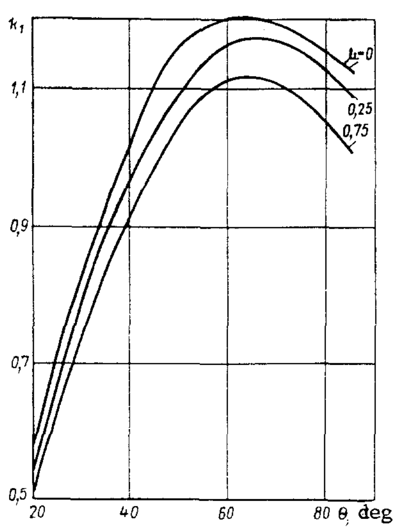


Fig. 2

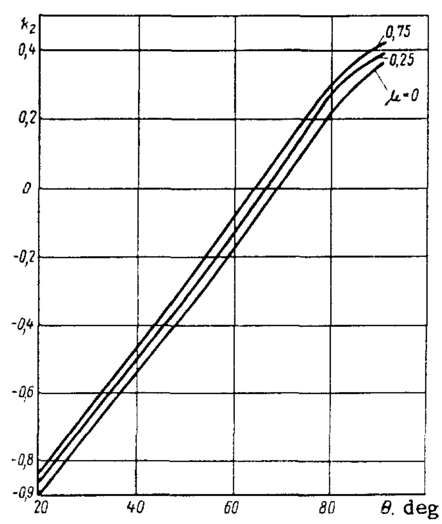


Fig. 3

It is established that $\theta_*$ is practically independent of the angle of application of the external load and decreases linearly with increase in frictional coefficient. This dependence is described by the formula

$$\theta_*(\mu) = 68^\circ - 5^\circ\mu.$$

The error of this dependence with respect to the results of numerical calculation is no more than 1%. The value of $\theta(+0)$ is close to the experimental data [5, 8]. Curves of $x = \pm c \pm \ell\cos(\theta(\ell))$; $y = \pm \ell\sin(\theta(\ell))$; $(0 \leq \ell \leq \ell_0)$ may give an idea of the form of the curvilinear side arms which appear. As follows from numerical analysis of the asymptotes for the two terms, the character of flexure of the developing crack in uniaxial compression ($\sigma_2 = 0$) depends on the angle $\gamma$. In the interval $\gamma \in (0; \pi/5)$, the results of calculation with $\mu = 0$ and $\tau_c = 0$ clearly show the downward convexity of the initial curvilinear crack; when $\gamma \in (\pi/4; \pi/2)$, upward convexity is seen.

## LITERATURE CITED

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