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THE EXPRESS METHOD OF DETERMINING THE FRACTURE TOUGHNESS OF BRITTLE MATERIALS

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Abstract. This paper presents a novel method of fracture toughness determination, - a method that does not require introduction of an initial crack or notch-type defect in a sample. The corresponding formula for calculating fracture toughness, which does not depend on crack length is derived. This “express” test is useful for industrial applications because it economical and may be used for a wide range of brittle materials from ceramics to concrete and rocks. Test results are shown to be in good accord with the results obtained by ASTM methods.

1. Introduction. Much work has been done on elaboration of laboratory test methods of fracture toughness determination for such materials as ceramics, concrete, rocks, etc. (for review, see Murakami (1987)). However, easily accessible and reliable test methods that industry needs for the material evaluation at wide range of temperature and environmental conditions seem to be missing.

The existing methods of measurement of fracture toughness (see, for example, Jaeger and Cook (1983), Nisitani and Mori (1985), Hertzberg R. (1983), and Staroselsky *et al*. (1990)) involve laborious process of introduction of a crack which initiates fracture. Furthermore, fatigue crack growth cannot be recommended for brittle materials, since it increases lengths of existing microcracks and other defects that change the properties of the sample.

Here, we suggest a method that eliminates these deficiencies. Fracture toughness determination is performed using samples without initial cracks. A sample has the shape of a disk with a circular hole in the center. It is monotonically loaded by a pair of compressive forces as shown in Fig.1. The central hole, playing the role of the defect, initiates fracture.

All existing methods for K_{IC} determination are based on elasticity solutions for bodies with cracks. Therefore, the corresponding expressions for K_I involve the initial crack length as a parameter. For the “crackless” sample and the loading scheme suggested here we obtain a formula for K_{IC} that depends only on the sample dimensions and on the critical value of the load.

The proposed method does not require any essential preliminaries to tests. It is particularly useful in industrial tests if the supply of material is limited.

2. Test Configuration and Mechanical Model. The basic idea behind the method is to eliminate the process of introduction of the initial crack. The sample (Fig. 1) is a ring with outer radius R_{out}, inner radius R_{in}, and thickness h that is sufficient to realize plane strain conditions. It is loaded by a pair of point forces P acting along diameter (AB). The distribution of the load across the thickness of the disk is uniform.

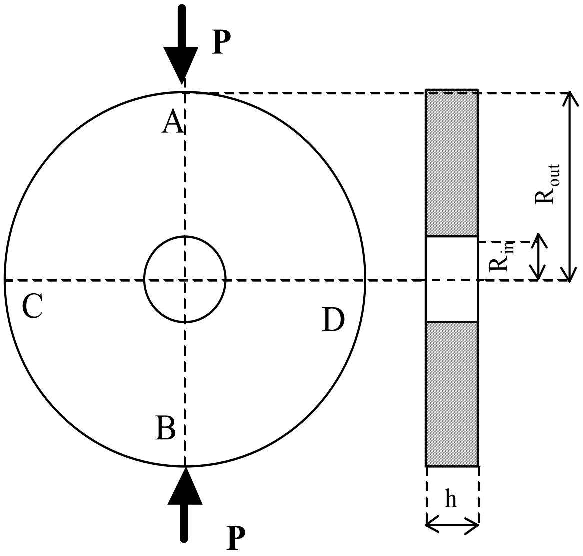

Figure 1. Test sample and loading scheme. R_{in} is the hole radius; R_{out} is the disk radius; h is the disk thickness, and P is the magnitude of applied load.

Friedman *et al.* (1972) show that drilling a hole creates a process zone (zone of microcracking) with a thickness of roughly one to two diameters of the average grain size appears around the hole. When the force is applied, the microcracks situated in close proximity to the line of the force (AB) at the edge of the inner hole start to grow and, at some value of the force, give rise to a macrocrack. Other pre-existing cracks within the specimen do not grow. A radial crack nucleates and propagates from the inner hole along the line of force (AB).

In the ring sample subjected to the compression along the diameter (AB) the normal stress component in the circumferential direction σ_θ is predominately tensile for the cross section (AB) and mostly compressive for the orthogonal cross section (CD). The corresponding elastic two-dimensional solutions for stress distribution in a solid ring have been obtained by Timoshenko and Goodier (1970) and Ripperger and Davids (1947) as an infinite series, and in different form by Muskhelishvili (1963). Because brittle materials are typically strong in compression and relatively weak in tension, failure starts at the point of the inner boundary along the diameter (AB).

To obtain the formula of fracture toughness for this load scheme, we take as a basis the approximate solution obtained by Murakami *et al.* (1986). Using the body force method they have considered two symmetrical cracks emanating from a hole in a ring under diametrical compression. They obtained the expression for stress intensity factor K_I useful for asymptotical analysis:

$$\mathrm{K}_{IC} = F(R_{in,}\ R_{out}, c) \cdot \frac{P}{\pi R_{out}\ h}\sqrt{\pi(R_{in}+c)}\,, \qquad (1)$$

Where P is applied load, R_{in} and R_{out} are radii of the hole and of the disk respectively, h is the sample thickness, c is the initial crack length, and F is the correction function, which Murakami *et al.* (1986) and Isida (1975) calculated numerically. The crack propagation is unstable because of the fact that derivative $\frac{\partial K_I}{\partial c} > 0$, and, subsequently, the fracture toughness may be determined from the geometrical parameters and the critical value of the applied force P.

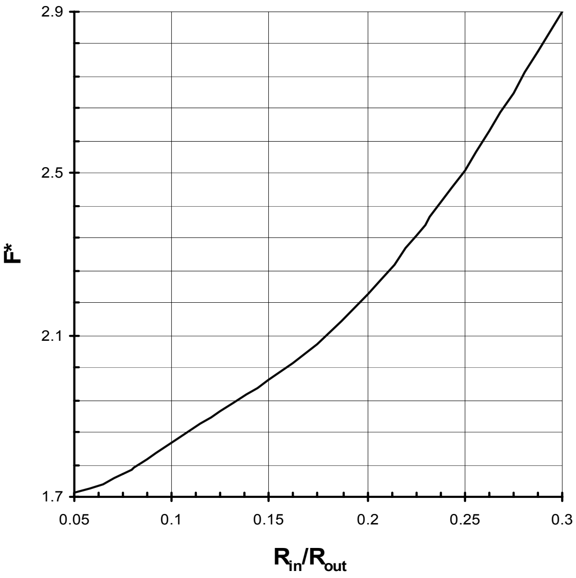

Figure 2. Non-dimensional correction function for fracture toughness

We asymptotically transform the shown expression by applying the fact that the initial crack length is much smaller than the radius of the hole. Namely, the actual crack length is about two average material grain diameters (see Friedman (1972). Multiple Taylor expansions of (1) with respect to small parameters $\frac{c}{R_{out}}$ and $\frac{c}{R_{in}}$ transform (1) into our calculation formula:

$$K_{IC} = F^{*}\frac{P\sqrt{R_{in}}}{\sqrt{\pi}\ R_{out}\ h}\left(1+\frac{d}{R_{in}}\right) \qquad (2)$$

where d is the average diameter of the grain. After asymptotical transformations the correction function F^{*} depends only on the ratio $\beta = \frac{R_{in}}{R_{out}}$; it is shown in Fig.

2. For most materials the second term in the parenthesis in (2) may be dropped. Thus the final expression for the fracture toughness is to be taken in the simple form:

$$K_{IC} = F^{*}\frac{P\sqrt{R_{in}}}{\sqrt{\pi}\;R_{out}\,h}. \qquad \textbf{(3)}$$

Note that dropping off the second term in (2), decreases calculated values of K_{IC} up to 5% depending on grain size and the sample dimensions. Increasing of F^{*} may compensate it.

3. Test Results and Discussion. Two sets of tests were conducted in order to evaluate the applicability and the accuracy of this method. The first set of fracture toughness data is obtained by our new scheme. We compare these test results with data from standard four point bending of prismatic samples with an edge crack (Murakami *et. al.* (1987), Jaeger and Cook (1983), and Staroselsky *et. al*. (1990)). We have tested disk specimens with radii varying from 21 mm to 36 mm. The thickness of disk specimens of 15 mm ensures plane strain conditions.

A picture of the solid sample after the test is presented in Fig. 3. As expected, in all our tests the crack develops symmetrically, beginning at the diameter of the inner hole away from the hole in the applied force direction.

Figure 3: The ring specimen after the test.
Two vertical symmetrical cracks have developed beginning at the edge the hole along the diameter in the applied force direction.

Figure 4: The ring specimen containing a horizontal pre-existing crack, destroyed by two vertical symmetrical cracks developing along the diameter.

It is often not possible to produce a solid specimen due to disruption of structures. Typical examples of this are layered materials, composites, and weak, cracked materials. We have analyzed the applicability of our method to the analysis of such materials. The sample shown in Fig. 4 contained a crack before testing. The crack length did not exceed the half radius of the disk. The load was applied in the direction perpendicular to this pre-existing crack. Experiments showed that the symmetrical vertical cracks in the direction of loading (Fig. 4) grow first. The horizontal (pre-existing) crack begins to develop only after the vertical cracks reach the outer disk boundary, i.e. the load is driven to the pre-existing crack at the moment of the destruction of the specimen, causing it to propagate. Thus, the possible existence of cracks in a ring sample does not influence the results. The method permits examination of materials, which are not successfully tested by standard methods, for example, weak, layered rocks, such as argillite.

The second set of tests was conducted according to standard ASTM four point bending method. Prismatic specimens with dimensions $90mm \times 30mm \times 15mm$ were used. Typical coefficient of variation for these tests did not exceed 10%.

The results of the tests together with the corresponding ratio $\frac{R_{in}}{R_{out}}$ for each ring test are given in the Table 1.

Table 1. The Fracture Toughness of Rocks

	Ring tests			4 points bending	
Rock Type	K_{IC} MPa $\sqrt{m}$	Var. %	$\frac{R_{in}}{R_{out}}$	K_{IC} MPa $\sqrt{m}$	Var. %
Aleurolite	0.95	12	0.11	1.01	10
Argillite	0.66	14	0.10	-	-
Granite	2.41	18	0.11	2.49	15
Diorite	2.34	16	0.19	2.39	14
Limestone	0.38	15	0.19	0.35	9
Marble	0.72	12	0.19	0.79	8
Scheelite Sulfide Ore	2.14	15	0.19	2.89	11
Sandstone 1	0.77	7	0.19	0.79	7
Sandstone 2	1.16	9	0.10	1.05	8
Skarn	2.38	16	0.19	2.46	8

Each test was repeated five times to ensure the reliability of the data. The values of coefficients of variation (Var. %) for the ring test data are higher than for the standard tests but nevertheless typical for rock mechanics experiments. They are usually higher in the case of weak pre-cracked rocks, such as argillite for which the standard methods failed to apply at all. Comparison of the two methods shows that the results differ no more than 10 - 15%.

Thus, the principal advantages of our express method, compared with known ones, are:

(*i*) wide area of the method application, - this method can be used to determine K_{IC} of a wide range of materials at different environmental conditions; (*ii*) no waste of time and labor is needed to prepare the experimental samples, - experimental samples are easily prepared and results are simple process; (*iii)* reduced usage of material (samples can be made out of a geological core).

Based on the above, the described "express" test method is a quick, useful, and reliable tool for determining K_{IC} for a wide class of brittle materials.

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