

THREE-DIMENSIONAL FATIGUE CRACKING UNDER ELASTIC-PLASTIC DEFORMATION

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Abstract

Aircraft engine structures often contain small cracks, which have developed from defects induced during material processing. Advanced structural materials such as nickel-based superalloys undergo extensive plastic deformation prior to failure, therefore, these small cracks can be subjected to extensive localized damage with significant amount of plasticity. We have conducted a combined computational / experimental study of fatigue crack growth at room temperature and at 260° C. The experimental results have been correlated with 3D FEM calculations. Material constitutive equations and a computational procedure to calculate energy release rate along the crack front are developed. It is shown that the fatigue crack growth rate is related to a power function of J_{\max} .

1. Introduction

The service life of structural components can be governed by several modes of degradation and failure such as fatigue, excessive deformation, fracture, yielding, creep, stress rupture, corrosion, wear, erosion, etc. One dominant mode of failure is due to fatigue, and structural components have to be designed so that they can adequately endure the fatigue loading during their service life. This paper focuses on the fatigue life crack growth prediction methodology for structural components, which are undergoing elastic-plastic deformation.

In aerospace applications, newer designs are operating at higher speeds and temperatures. Consequently, there are regions of notch plasticity which are contained or surrounded by elastic fields. Situations may also arise where a large region, especially in smooth areas, undergoes elastic plastic deformation. This is a result of a combination of thermal and

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mechanical strains that the component experiences in its flight or service cycle. Fatigue life estimations for such components, are typically performed, by assuming a flaw size and determining remaining life based on crack growth predictions. The initial flaw, may correspond to the smallest inspectable flaw size, or to an initial quality defect such as an inclusion cluster. Crack growth predictions from "inspectable size" flaw(s) give an estimate of remaining life; crack growth predictions from "initial quality" flaw(s) give an estimate of total life. When there is appreciable elastic-plastic deformation, use of the conventional fatigue crack growth technique of Linear Elastic Fracture Mechanics (LEFM) can lead to overestimates of fatigue life (Figure 1). This is due to the breakdown of the basic LEFM assumptions.

The consideration in LEFM is that the plastic deformation near the crack tip is small compared with the crack length and other relevant dimensions of the structural component, and that the elastic singular fields dominate asymptotically. A simple model that captures the additional zone of plastic deformation when LEFM breaks down is the Dugdale-Barenblatt (DB) Model [1]. The DB model which is used for Mode I or the crack opening mode is complemented by the Bilby-Cottrell-Swinden (BCS) [1] model which is for Mode II or pure shear loading. In both cases, a yield strip is used at the end of the physical crack increasing its "effective" length. Yield strip and contact models capture the plastic wake along the crack surfaces. Most of the existing methods for life prediction are based on small-scale plasticity approaches.

However, for gas turbine engine applications, the nature of plastic zones can be very diffuse and spread over a large region, for example, in the bores of compressors / turbine disks. It means that the size of the plastic zone is large in comparison to the crack length, so the stress intensity factors or other linear fracture mechanics parameters loose their meaning and cannot be employed in predicting crack growth rate. This has led to a number of approaches, of which the most established is the energy release rate concept and its mathematical formalism - the path-independent J-integral:

$$J = \int_{\Gamma} \left(W n_1 - T_m \frac{\partial u_m}{\partial x_1} \right) ds .$$

Here, W is strain energy density, n_1 is x_1 component of unit

normal to the contour Γ , $T_m = \sigma_{mk} n_k$ is the traction, and u_m is the displacement component. As already has been shown [1], J-integral does not retain path-independence

under unloading, so its calculation during unloading is not meaningful in a strict mechanics sense. One may compute the cyclic energy release rate

$$\Delta J = \int_{\Gamma} \left(\Delta W_{n_1} - \Delta T_m \frac{\partial \Delta u_m}{\partial x_1} \right) ds, \text{ and try to fit the Dowling modification of the Paris}$$

equation [5]: $\frac{da}{dN} = C(\Delta J)^n$, where C and n are fitting parameters. It is important to stress

that the approach purely based on ΔJ -concept has been verified only for moderate plastic zone sizes, does not take into account the mean stress and usually deviates from test results. Tanaka [2], Banks-Sills and Volpert [3] showed that this deviation increases with the spread of plastic zone across the specimen cross-section. In addition, this ΔJ calculation is a laborious procedure, which depends on deformation history over each cycle.

The paper is organized as follows. In the next section a fatigue crack growth rate constitutive model is developed. In section 3, the finite-element method is used to determine the energy release rate. The fourth section deals with the experimental testing of nickel super-alloy material with surface cracks and determination of parameters for crack growth. A summary and conclusions are presented in Section 5.

2. Crack growth rate constitutive law and governing equations

The aim of this paper is to propose a simple and accurate expression for crack growth rate prediction. First, for simplicity, cracks closure effects are neglected and our consideration is limited to tension-tension cycles analysis. The fatigue crack is driven by the cyclic plastic deformation at the crack tip, and it propagates by sequential blunting and re-sharpening (Riemelmoser et al. [4]). Thus, under tension, the crack initially opens and blunts, then, if tension increases, the crack may start propagating. The crack growth during this load cycle is accompanied by striation formation¹. It is noted that the fatigue crack

growth for positive R-values, (where $R = \frac{\sigma_{\min}}{\sigma_{\max}}$) is primarily determined by the maximum tension strain. Based on this assumption, the fatigue crack growth can be governed by an envelope of cyclic strain-stress relations, which is a function of the single parameter J_{\max} .

¹ This description of fatigue crack growth, certainly, is oversimplified here.

Also, for small positive stress ratios $0 \leq R \leq 0.2$ the minimum values of energy release rate are much smaller than the maximum ones, $J_{\min} \ll J_{\max}$, therefore, in the first approximation $\Delta J \sim J_{\max}$. Thus, combining this reasoning with the Paris-Dowling [5] power law representation, we propose a fatigue crack growth rate constitutive relation in the following form:

$$\frac{da}{dN} = A J_{\max}^q, \quad (1)$$

where A and q are fitting parameters. To account for various stress ratios, the relation may be modified as shown below. Following similarity with linear fracture mechanics and taking into account that $\Delta K = K_{\max} - K_{\min} \equiv K_{\max} (1 - R)$ we may express the crack growth rate as the following power function of J_{\max} and R-value: $\frac{da}{dN} = A' J_{\max}^q (1 - R)^m$. In this paper, the analyzed test results are for only one value of R, therefore, equation (1) is used to fit test data, setting $A = A'(1 - R)^m$.

The computation of J (and ΔJ) involves advanced finite element analysis and the result is a strong function of the material elastic-plastic constitutive behavior, strain hardening, in particular. In our work, two typical material behaviors are analyzed: (i) elastic-perfectly plastic stress-strain response, which corresponds to Ni-based superalloy behavior at low homologous temperatures and (ii) a power-law hardening material or the so-called Ramberg-Osgood relation used in the Hutchinson-Rice-Rosengren (HRR) model. The Elastic-Plastic constitutive relation within the framework of small strain theory can be developed based on the decomposition of the total strain tensor as follows:

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \quad (2)$$

where ε_{ij}^e is the elastic strain part and ε_{ij}^p is the plastic strain part. It generalizes as follows:

$$\varepsilon_{ij} = \frac{1 + \nu}{E} s_{ij} + \frac{1 - 2\nu}{3E} \sigma_{kk} \delta_{ij} + \frac{3\alpha}{2E} \left(\frac{\bar{\sigma}}{\sigma_0} \right)^{n-1} s_{ij} \quad (3)$$

where ν is a Poisson ratio, E is Young's modulus, $s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$ are the deviatoric stresses, and $\bar{\sigma}$ is the effective stress. Parameters α and n change with number of cycles due to cyclic isotropic and kinematic hardening. By using comparison between material model prediction and measured stress-strain relations for a series of snapshots, information

about parameters α and n is deduced for corresponding cycles N . Fatigue crack size $a(N)$ and crack growth rate $\frac{da}{dN}$ are also measured at the same time. We assume in our model that crack growth is self-similar, and crack continues to be semi-circular in shape. With the material model so calibrated, values of J_{\max} are calculated numerically. The results of computations are matched with experimentally measured crack growth rate $\frac{da(N)}{dN}$ and the model (1) is calibrated by fitting parameters A and q . In the next section we discuss details of the numerical procedure.

3. Numerical Modeling

A nonlinear finite element analysis is used to solve a boundary value problem for planar, surface, and embedded cracks in three-dimensional elastic-plastic media. In this research effort, the method of virtual crack extensions by D. Parks [6] is used to compute the non-linear crack growth parameters such as the energy release rates. We analyze the fatigue crack growth conditions and energy release rate in the vicinity of surface semi-elliptical crack. This type of flaw shape is most commonly observed in structural components such as turbine disks. The crack with the depth a and the width $2c$ is embedded in a plate of thickness t and the width $2w$. The height of the plate is chosen to eliminate any boundary effects. Model dimensions correspond to actual sizes of tested specimens. Due to geometric and loading symmetry in the problem, only a quarter of the specimen is considered. Cracks with various aspect ratios and depths are modeled. An example of a finite element mesh using the MARC finite element code is shown in Figure 2. We generally use the mesh with a thousand of 20 node brick elements. The “spider web” element configuration is created at the crack tip region. The crack tip region consists of concentric rings of elements that are focused towards the crack tip. The elements of inner most-ring are collapsed into wedges. In this configuration, eight nodes occupy the same spatial point at the crack tip but move apart as the elements deform. This element geometry produces a $\frac{1}{r}$ strain singularity, which is an appropriate configuration

for performing elastic-plastic analyses². This element design also allows computing the crack blunting and the crack tip opening displacement (CTOD) for the deformed mesh as schematically shown in Figure 3 [7].

The J-integral is calculated using the virtual crack extension method available in the MARC finite-element code. This procedure evaluates the J-integral by calculating the change in strain energy due to an applied small nodal movement around the crack tip and by normalizing this energy difference to the crack extension area.

The FEM results have been verified against elastic solutions by Raju and Newman [8] and the agreement is within 3%. Next, we analyze the relation between J and applied far field strain for an elastic-perfectly plastic material. The typical results are shown in Figure 4 for different points along the front of a penny-shaped crack. We measure polar angle ϕ from the surface crack point. There are three different regions distinguishable on the graphs.

First is a quadratic curve reflecting elastic behavior. The second region corresponds to the elastic-plastic transition with a specific “elbow” form. Due to stress concentration and essential triaxiality next to the crack, the transition starts at approximately at $0.9\varepsilon_y$, and turns into fully plastic region at $1.05\varepsilon_y$, where ε_y is the material yield strain. The third segment of these graphs is a straight line, which reflects fully plastic material behavior. In order to understand the relation $J(\varepsilon)$ and elastic-plastic transition effect, we plot the

derivative $\frac{dJ}{d\varepsilon}$ versus applied strain ε as shown in Figure 5. One may see from the Figure

that elastic-plastic transition corresponds to a significant jump in J-integral rate and it reaches the maximum exactly at yield strain. After reaching the maximum point the rate appears to stabilize to a constant value, however, some drop in the $\frac{dJ}{d\varepsilon}$ values is observed

only for near surface points of the crack front³. This behavior depends on the crack depth and changes in plastic constraints ahead of the crack front, or in other words, the

² In the HRR model, the stress and strain have different singularities depending on the hardening exponent.

³ We did numerous computational experiments to figure out the cause of this $\frac{dJ}{d\varepsilon}$ drop. It turns out that for shallow surface cracks such a drop is omnipresent. The drop starts to disappear for crack points located deeper than a quarter of the specimen thickness.

difference in plane-stress and plane-strain yielding. An explanation of the role of plastic constraint is given in the Appendix.

The loading conditions are analyzed when the specimen cross-section is near the plastic limit. Therefore, even small stress concentration causes gross yielding. The numerical analysis has shown that the elastic-perfectly-plastic solution strongly depends on specimen geometry and crack sizes. This is, again, the manifestation of plain strain - plane stress transition. The plastic zone initiated by relatively short crack spreads out along the free surface. In contrast, the plastic zone induced by the deeper crack is localized around the crack front and significantly increases the “effective crack” size. With the size of the plastic zones so considerable, the crack – free surface and crack - crack interaction effects become notable at the distance two-three times larger than it is observed in LEFM. In order to illustrate the principal difference in the elastic and plastic crack sensitivity we plot

in Figure 6 the normalized values of $J(a/t)/J(0.1)$ calculated at the crack deepest point for both types of material behavior. It is clear from the Fig. 6 that in load – control scheme, the elastic-plastic J “blows up” and, subsequently, the crack strongly interacts with free boundary when its length reaches the half of the specimen thickness. In contrast, the elastic crack manifests almost linear behavior up to significantly larger crack sizes (Fig. 6) without noticeable crack-surface interaction.

The local normalized J-integral values $\left(J(\phi)/J(0) \right)$ along the crack front starting from the surface point are given in Figure 7. The solution for elastic - perfectly plastic material significantly differs from the results for hardening material, especially around the crack deepest point. J-integral predictions for elastic-plastic HRR material are close to the results of Yagawa et al [9] for fully plastic materials if the amount of plastic deformation is significant and exceeds ~0.5%.

4. Material Testing

A surface cracked rectangular specimen, which is 5.84 mm thick, 13.5 mm wide, and 40 mm long, have been used for the fatigue crack growth analysis. Initial surface flaw of 0.64 mm. was induced by electric discharge machine (EDM). The initial crack is obtained by

fatigue growth from the EDM notch at room temperature. All high temperature tests have been conducted at typical turbine disk bore temperature of 260° C.

In an effort to measure crack size and surface strain values when performing tests, strain gages, extensometry and a video monitoring system have been employed. Strain gages include both far field and near field measurements such that the elastic and inelastic strains can be separated. Video images of the surface crack are stored on tape for post mortem correlation to fatigue crack striation band markers obtained using a scanning electron microscope as shown in Figure 8a. Figure 8b shows the details of fracture surface at large magnification, which are typical for plastic crack propagation. We use these

measurements to evaluate the left side of the relation (1) - $\frac{da}{dN}$. In order to compute J-values for corresponding cycles we monitor stress-strain loops. Choosing only loading part of the loop, the Ramberg-Osgood parameters are fitted to the experimental results. It is important to note that due to the material hardening, these parameters $\alpha(N)$ and $n(N)$ change with loading history and are determined as functions of number of cycles. The calibrated HRR material law has been used in numerical calculations of J-values. The experimentally measured crack growth rate versus numerically calculated J_{max} is shown in Figure 9 in log-log coordinates. The model parameters have been estimated by least square method and are $A = 4.0E - 8$ and $q = 1.035$ for the value of $R=0.15$.

It is important to note that LEFM based measurements typically overestimates the critical crack resistance. There are two principal types of the fracture mechanics tests. As we discussed above the extensive plastic zone significantly affect the observed measurements. First one is the analysis of large cracks in large specimens under small elastic strain conditions, which agrees with LEFM predictions. The second type is the testing of small cracks in plastically deformed specimens, which is the subject of this paper. The results of static tests to the failure of specimens with crack sizes not exceeding 1.78 mm show

$J_{IC} = 44.3 \frac{kJ}{m^2}$ for a powder superalloy that has been analyzed. This is almost a half of the value of G_{IC} obtained in LEFM tests for the same material.

5. Concluding Remarks

A computational and experimental study of fatigue crack growth in inelastically deformed metals has been presented. Detailed finite-element models have been used to carry out energy release rate computations. The parameters for the Ramberg-Osgood power law material constitutive model have been obtained by fitting the stress-strain data obtained from tests. The J-integral has been evaluated for different crack sizes and loading conditions. The fatigue crack growth rate model based on a J_{\max} approach has been developed and verified. Parameters for this LCF model have been obtained from fatigue test data. The results clearly demonstrate the ability of the model to predict the crack growth rate under elastic-plastic deformation conditions. The effect of high crack sensitivity to specimen geometry in the elastic-plastic regime has been demonstrated. The drop in the fracture resistance is clearly shown in this case.

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Appendix

Following Turner [10] we use limit analysis for energy estimation. It is applicable for rigid plasticity and provides a qualitative explanation for plane-stress plane-strain transition in plastic regime. The limit load in tension is $F_L = C(a)B\sigma_y b$, where $b = w - a$ is a ligament, B is the thickness, and $C(a)$ is the plastic constraint factor. We assume here that it is a function of crack depth. Potential energy is proportional to $U_{pl} = u F_L$, where u is

displacement. Using the definition of J-integral, $J = -\frac{1}{B} \frac{\partial U_{pl}}{\partial a}$ immediately obtain,

$\frac{\partial J}{\partial \varepsilon} = -\frac{H}{B} \frac{d F_L}{da}$, where H is the specimen height. Substituting here the expression for

limit load, we finally get the relation between the slope of $J(\varepsilon)$ and the constraint factor as

follows: $\frac{d J_{pl}}{d \varepsilon} = \alpha \frac{F_L}{Bb} \left(1 - \left(\frac{b}{C} \right) \frac{dC}{da} \right)$. For relatively shallow cracks, the constraint factor is

a strong function of crack depth, asymptotically saturating for cracks with sizes of

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Figure 3: Deformed finite element mesh at crack tip [7].

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Figure 5: $dJ/d\varepsilon$ versus applied far-field strain, $a = 2$ mm, $W=13.5$ mm $t = 5.84$ mm

Figure 6: Normalized J – values for a surface semi-circular crack at its deepest point for linearly elastic and elastic-perfectly plastic materials.

Figure 7: Comparison of J-integral values along the crack front for an elastic-perfectly plastic and a work hardening material

Figure 8: (a) Scanning electron micrograph showing fatigue crack striation marker banding, (b) detailed view of fracture surface

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