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USE OF HEAT GENERATION AT CRACK TIPS FOR THE DETECTION OF HIGH CYCLE FATIGUE IN STRUCTURES

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Abstract. This paper presents the basis for the detection of small cracks using thermal non-contact techniques. The analytical expressions for the intensity of heat generated in crack tips plastic zones during each loading cycle and for corresponding structural temperature rise are presented. It is shown that this temperature rise can be detected at distances up to ten crack sizes.

1. Introduction. Damage detection is an important structural health-monitoring problem, which ideally should identify cracks at an early stage. In usual displacement measurements, a crack will disturb the field around it at distances on the order of the crack size; therefore the defects would be detected only when their size had already passed a critical stage. This makes the inspection process very tedious, if not impossible. Alternative approaches should analyze other measurable physical fields that do not vanish as quickly as the strain. In this discussion we focus our attention on the steady-state temperature distribution in the structure around cracks as a means to detect cracks, using scanning non-contact techniques. Indeed, since fatigue cracks periodically open and close and crack tips generate plastic deformation there will be heat added during each loading cycle, and this heating will result in a temperature rise over the nominal structure temperature. We analytically show that the temperature rise due to this source may be large enough to be detected by an infrared camera.

2. Physical background of the problem. The physical hypothesis for our model is as follows: (1) heat is generated in plastic zones surrounding crack tips; (2) all dissipated energy from the plastic zones is converted to heat. We intentionally simplify the problem ignoring crack surface friction and other effects in order to obtain conservative estimates. We also assume that fatigue crack growth is insignificant during the observation time. In order to predict the temperature distribution, the intensity of the heat generation should be expressed through measurable fracture mechanics parameters.
Energy dissipation is a strong function of the material elastic-plastic behavior as well as of the shape and the size of the plastic zone around crack tips. In this work

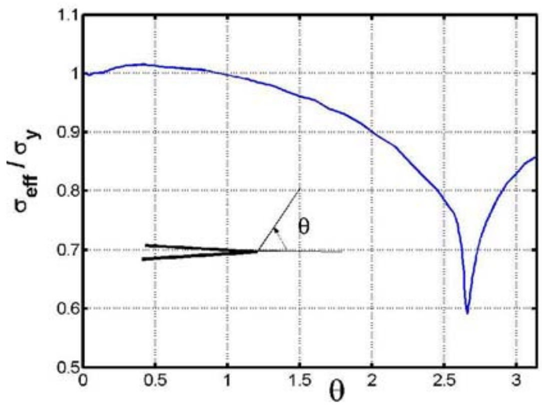

Figure 1. Normalized stress around crack tip for HRR model n=13 (Hutchinson, 1979).

we assume a power law hardening material behavior according to Hutchinson-Rice-Rosengren (HRR) model. This model accurately represents the material response of a wide class of metals and alloys, in particular used in the aerospace industry. Plastic deformation within the framework of small strain theory has the following general form: $\varepsilon_{ij}^{p} = \frac{3}{2}\alpha\varepsilon_y \left(\frac{\overline{\sigma}}{\sigma_y}\right)^{n-1} \frac{S_{ij}}{\sigma_y}$, where ε^p_{ij} and s_{ij} are plastic strain and deviatoric stress tensor components and $\overline{\sigma}$ is the effective stress, and σ_y is the yield stress. Parameters α and n are specific for the material and should be determined experimentally. Since there is a power law singularity at the crack tip (Kanninen and Popelar 1985) one might neglect small elastic strains around the crack tip. It is known (Hutchinson 1979) that for HRR materials the asymptotic field around the crack tip has the following form: $\sigma_{ij}(r,\theta) = C\, r^{-1/_{n+1}} \tilde{\sigma}_{ij}(\theta)$; $\varepsilon_{ij}(r,\theta) = \alpha\varepsilon_y C^n\, r^{-n/_{n+1}} \tilde{\varepsilon}_{ij}(\theta)$; and, subsequently, the energy per unit volume in the plastic zone is $\sigma_{ij}\varepsilon_{ij} \sim \frac{f(\theta)}{r}$. In the asymptotic field, C is a constant, r -is the distance from the crack tip and θ is the polar angle. Having established the stress-strain distribution, the expression for the J-integral can be written as $J = \alpha\varepsilon_y\sigma_y C^{n+1} I_n$. Numerical values for I_n are given in (Hutchinson 1979) as well as values for stress around the crack (see Fig 1.) Eliminating the constant ***C*** we obtain the relationship for the energy density around the crack tip: $W(\theta,r) = \frac{1}{I_n}\frac{n}{n+1}\cdot\left(\frac{\tilde{\sigma}(\theta)}{\sigma_y}\right)^{n+1}\cdot\frac{J}{r} = \frac{\hat{W}}{r}$ and to the first approximation, the power for cyclic loading can be expressed as $\dot{W} = \frac{\hat{W}}{r} f$, where f is loading frequency. Since the observed temperatures will be far from the crack tips, we will use average values over the polar angle to evaluate the energy. Then denoting the averaged value needed for energy estimation through $\kappa_n = \frac{n}{n+1}\cdot\frac{1}{\pi}\int_0^{\pi}\left(\frac{\tilde{\sigma}(\theta)}{\sigma_y}\right)^{n+1} d\theta$

and $\chi_n = \frac{\kappa_n}{I_n}$, and using the data from Fig 1, we obtain the general expression for the power of plastic dissipation in the general form $\dot{W} = \chi_n \cdot \frac{\bar{J}}{r} f$, where $\bar{J}$ is the averaged J-integral over a loading cycle. For low hardening materials n=13, (a fit to Ni-superalloys), and, subsequently, I_{13}=2.87 (Hutchinson 1979), $\kappa_{13} = 0.1835$, and $\chi_{13} = 0.065$. For harmonic loading $\sigma = \sigma_0 + \sigma_m \sin(2\pi f t)$ the average value of J is $\bar{J} = \frac{\pi a}{2E}\left[\sigma_0^2\left(1 + \frac{1}{2}\left(\frac{\sigma_0}{\sigma_y}\right)^2 + \left(\frac{\sigma_m}{\sigma_y}\right)^2\right) + \frac{\sigma_m^2}{2}\left(1 + \left(\frac{\sigma_0}{\sigma_y}\right)^2 + \frac{3}{8}\left(\frac{\sigma_m}{\sigma_y}\right)^2\right)\right]$, where a is the half-length of a crack and E- is Young's modulus.

We assume to first order that the plastic zone can be approximated as a circle around the crack tip. In hardening materials, assuming plane stress, the radius of the circle can be determined from the maximum applied stress (Kanninen and Popelar 1985) as $r_p = \frac{\sigma_{\max}^2}{2\sigma_y^2 - \sigma_{\max}^2} a$. It is inside this circle that the energy will be generated. We assume that during cyclic unloading the plastic zone does not shrink. The characteristic size of such a zone for a 1 mm crack varies with the ratio $\sigma_{\max}/\sigma_y$ from 25 – 50 μm for a typical superalloy blade up to 0.5 mm for soft aluminum parts. In the first case, we may approximate the heat generation by a point source; for the second one, we have to consider the distributed plastic zone and, subsequently, the distributed heat source. In this paper we present the analysis for both point and distributed heat sources.

3. Statement of the Problem. We consider the steady state temperature distribution in a thin fin, which models a turbine or fan blade, containing a small central crack. The fin is subjected to cyclic tension-tension loading, which eliminates crack tip closure. Crack tip plastic zones generate heat with the intensity of each heat source given by $\dot{W} = \frac{\hat{W} f}{r}$. Using the linearity of the problem, we solve for the axisymmetric temperature distribution from each source and superimpose the solutions. Using cylindrical coordinates, with the origin at the crack tip, the problem is reduced to one-dimensional heat transfer equation with the distributed heat source inside the plastic zone:

$$\frac{d^2\vartheta}{dz^2} + \frac{1}{z}\frac{d\vartheta}{dz} - \vartheta + q = 0, \qquad q = \begin{cases} 1/z & \text{inside plastic zone} \\ 0 & \text{outside plastic zone} \end{cases}$$

where $\vartheta = \frac{T - T_a}{\hat{W}f / \sqrt{\kappa h}}$ is the dimensionless temperature, κ is the fin thermal conductivity; $h = \frac{2H}{t}$; H is the convection heat transfer coefficient, t is the fin thickness, $\alpha = \sqrt{h/\kappa}$, and $z = \alpha r$ is the dimensionless distance. Outside the plastic zone the heat source is absent. The boundary conditions are as follows: ϑ is finite at $z = 0$, and we set $\vartheta = 0$ at $z \to \infty$. Additional conditions are required on the plastic zone boundary. Since heat is generated due to plasticity the temperatures and heat fluxes at the outer boundary of the heat source-border of plastic zone, must satisfy: $\vartheta_1 = \vartheta_2$; $d\vartheta_1/dz = d\vartheta_2/dz$ @ $z = z_p$. Solution of the problem with a distributed heat source is needed for materials/loading schemes with diffused plastic zones. In case of a small plastic zone, the plastic zone can be readily substituted by a point source with the intensity $Q = 2\pi \int_0^{r_p} \frac{\hat{W}}{r} r dr = 2\pi r_p \hat{W}$ - equivalent to the distributed in the plastic zone case. In the heat transfer equation for a point source we just omit the "q" term, putting heat source in the boundary condition. For a point source at the origin, the boundary conditions become: $\begin{cases} \vartheta = 0 & @\ z \to \infty \\ \frac{d\vartheta}{dz} = -1 & @\ z \to 0 \end{cases}$. The first condition assumes no change in temperature far from the crack tip, while the second assumes that the heat is uniformly generated on a line throughout a thickness along the crack tip and insures that the total heat output from the line source enters the fin.

Based on a dimensional analysis, the temperature distribution in this case may be written as $T - T_a = \frac{Q}{2\pi\kappa}\theta^{point}(z)$, where $\theta^{point}(z)$ is the normalized solution of the problem with the point heat source. Substituting the expressions for Q and for $\vartheta(z)$, one obtains that $\frac{T - T_a}{\hat{W} f / \sqrt{\kappa h}} \equiv \vartheta(z) = z_p \theta^{point}(z)$, where z_p is normalized radius of plastic zone.

4. Modeling and results of simulation. First consider the axisymmetric problem with a point heat source. The solution is $\theta^{point} = K_0(z)$, where K_0 is the modified Bessel function of second kind. Taking into account the expression for the equivalent heat source and the fact that both crack tips generate heat, the solution has the following form:

$$\vartheta = z_p \cdot \left\{ K_0\left(\alpha\sqrt{(x-a)^2 + y^2}\right) + K_0\left(\alpha\sqrt{(x+a)^2 + y^2}\right) \right\},$$

where $r = \sqrt{x^2 + y^2}$ and $2a$ is a crack length.

The solution of the problem with distributed source can be obtained by variation of parameters from the solution for the point source. After applying the boundary and matching conditions the particular solution for one source is:

$$\vartheta = \begin{cases} c_1 I_0(z) + F(z) & z < z_p \\ c_2 K_0(z) & z > z_p \end{cases} \qquad F(z) = K_0(z)\int_0^z I_0(\varsigma)d\varsigma + I_0(z)\int_z^\infty K_0(\varsigma)d\varsigma$$

For the general case, coefficients depend only on the plastic zone size and can be determined from the expressions below:

$$c_1 = z_p\left[-I_0(z_p)F'(z_p) + I_1(z_p)F(z_p)\right]; \quad c_2 = -z_p\left[K_0(z_p)F'(z_p) + K_1(z_p)F(z_p)\right]; \quad F' = \frac{dF}{dz_p}.$$

However, the size of plastic zone is usually small and for this case the coefficients asymptotically become equal to $c_1 = -\frac{\pi}{2} - z_p \ln z_p + (\ln 2 + 1 - \gamma)z_p + O(z_p^2)$; and $c_2 = z_p + O(z_p^3)$; as $z_p \to 0$. One may see that the temperature distribution outside the plastic zone coincidences with the solution for point heat source located at the crack tip. Results of temperature calculations for typical Ni-based superalloy material parameters, and for natural convection, are given in Figure 2 for different mean stresses with an oscillating magnitude of 15 ksi. The yield stress is equal to 100 ksi. A frequency of 1 kHz was used, corresponding to typical high cycle fatigue loading. One may see from this solution that temperature changes of

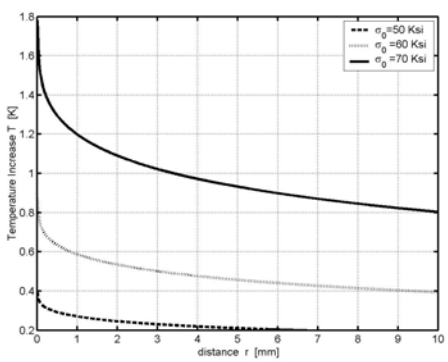

Figure 2. Cyclic-Crack tip induced temperature in front of the central crack with the length of 2a=1 mm.

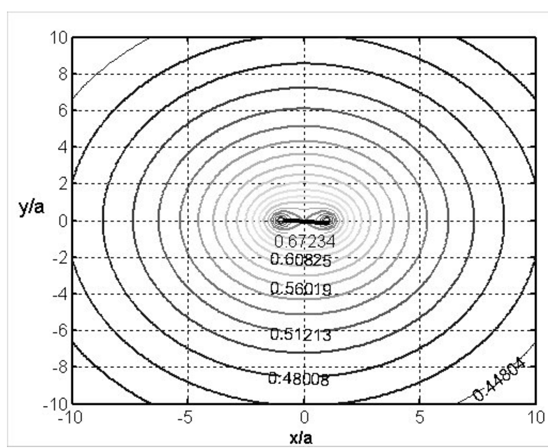

Figure 3. Plastic-dissipation induced temperature contour lines for the central crack.

0.2 degree may be detected over at least ten crack sizes.

We have assumed throughout the fact that the volumetric heating over time is given by the total number of cycles, which has important consequences in high cycle fatigue. For typical turbine engine components the natural frequencies of importance in design can be as high as 20 kHz. Our example of 1 kHz is conservative in the effects of heating, but is indicative of the heating for the more important low frequency modes and is also near the limiting capacity of laboratory testing machines. In the example presented in Fig. 2, we assumed cooling was by natural convection. For forced convection the heat generated at the crack tips will be readily removed, and hence will have less effect on the temperature rise. Rough calculations demonstrate that the heat transfer coefficient for forced convection in a turbine engine compressor will increase by about one hundred times over that for natural convection. Since the distance is scaled by the square root of the convection coefficient, this means that distances will shrink by an order of magnitude in Fig. 3. Even though there is a significant reduction in the distance, it should still be larger than the distance over which the strain distribution can be observed, since the loading frequency is typically higher than 1KHz as was noted above.

5. Conclusions. We developed a physical model relating fracture mechanics parameters and energy dissipation in the plastic zone with temperature rise in the elastic-plastic structure. The current model provides basic axisymmetric solutions and can be easily generalized for different types of plastic yield, for example, described by Dugdale model.

The temperature rise caused by a fatigue crack is distributed over a range wider than the strain distortion and, hence, may be detected using existing non-contact scanning technologies and hardware. This approach should be especially useful in high cycle fatigue situations. Although the technique is most useful in low convection heat transfer, it should still be applicable even in cases with high convection heat transfer present. The work, which has been done to date, clearly demonstrates the feasibility of damage diagnostics based on thermal field measurements. Existing test results (Favro *et al*. 2001) demonstrate feasibility of this structural health monitoring methodology, however, additional experiments need to be performed to clarify the bounds and limitations of the technique.

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