

**ACOUSTICALLY INDUCED MASS FLOW IN PARTIALLY SATURATED POROUS MEDIA. APPLICATIONS TO FUEL CELLS.**

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**Dr. Alexander Staroselsky**

Pratt & Whitney,  
400 Main Street, M/S 165-16,  
East Hartford, CT 06108 USA  
Phone: (860) 565-2751; fax: (860) 755-5511  
e-mail: Alexander.Staroselsky@pw.utc.com

**Dr. Igor I. Fedchenia**

UTRC, 411 Silver lane,  
East Hartford, CT 06108  
fedcheii@utrc.utc.com

**Dr. Wenlong Li**

Mississippi State University  
Department of Mechanical Engineering  
li@me.msstate.edu

**ABSTRACT**

In this work we aim developing a novel theoretical framework to evaluate the feasibility of attaining significant improvement of fuel cells performance and stability by increasing the transport processes in porous partially fluid filled cathode compartment through the application of acoustic and structural waves. We have developed a generic unified model of structural/acoustic wave propagation in the porous media and coupled it with mass transfer. It has been demonstrated that the phase saturations have strong impact on wave dynamics in porous continuum. Explicit expressions for generalized multiphase Biot-type coefficients have been obtained. A novel generalized filtration law that accounts for dynamic loadings, varying saturation, and solid structure distortion describing mass transfer in this complex but generic system has been found. The Lagrangian approach has been employed in order to develop consistent governing equations. We have also conducted highly precise tests on flow measurements throughout porous media with and without applied acoustic signals allowing the model calibration.

It has been demonstrated that vibration gives rise to net change of saturation inside porous medium and that applied structural vibration/acoustic loading intensifies the process dynamics. Based on the numerical and experimental results number of practical recommendations optimizing material selections and performance regime has been made.

**INTRODUCTION**

The basic physical structure, or building block, of a fuel cell consists of an electrolyte layer in contact with a porous anode and cathode on either side. The cells run on hydrogen, which reacts with oxygen from the air in such a way that a voltage is generated between two electrodes. The electrolyte in this Fuel Cell is the ion exchange membrane (Nafion or other similar polymers) and is a proton ( $H^+$ ) conductor. The electrons flow to the external circuit generating current.

Transport and reaction processes at the cathode limit PEMFC performance. Oxygen present in the air stream flowing through the channels in the so-called bipolar plate (BPP), is transferred to the cathode, through a porous diffusion layer. Water produced by the electrochemical reaction at the cathode is partially evacuated through the same porous medium to the air stream.

Limitations of PEM cell performance at useful current densities ( $\geq 0.5 \text{ A/cm}^2$ ) are primarily dominated by mass transfer of water. In order to deploy PEM fuel cells in high power density applications (e.g. automobiles and ships), stable performance must be achieved at these high current densities and in turn to obtain high current density, oxygen must be transported to and water removed from the electrochemically active catalyst sites. In a sense, the necessity for water removal comes from its role

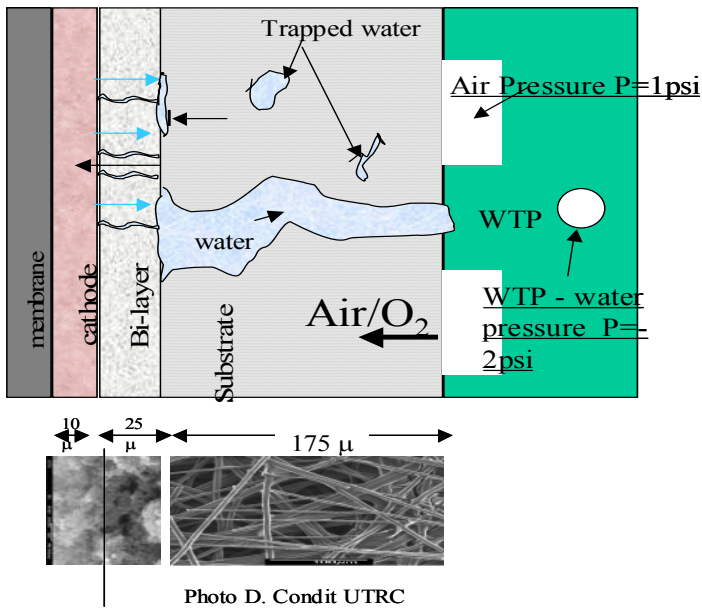


Figure 1. Structure of Fuel Cell Cathode compartment and directions of major mass flows.

in inhibiting mass transfer of oxygen. Since the PEMFC operates at a low temperature, water hang up (known as cathode flooding) in a liquid state is unavoidable. This not only impacts PEMFC performance but also leads to performance instability [1]. Ideally, the water that has been produced at the cathode is driven to the air channel where the air would entrain excessive liquid water. At the same time this airflow would supply the necessary oxygen for the fuel cell. Thus, the directions of oxygen and water flows are opposite to each other.

In this work we analyze the feasibility of trapped water removing and increase of the material effective permeability by application of structural waves to the system. There have been a number of works analyzing waves propagation in fully saturated porous media [2, 3] – so called Biot models. All known to us publications study the influence of properties of porous materials and fully saturated them fluids on waves characteristics. In contrast, we study wave propagation and attenuation in partially saturated porous media and extend Biot’s approach for multiphase (solid, fluid, and gas) media. Also, we develop a novel approach combining both “elastic-wave” part and “viscous – mass transfer” part coupling transport and acoustic properties. In other words we develop a generic model allowing us to evaluate water and gas transfer induced by structural waves. We calculate characteristics of different waves in the structure and couple them with water/gas pressures and induced inhomogeneous in space and time water saturation level–  $S(x,t)$  and, also, relate it with oxygen transport answering the practical question – could the structural waves increase oxygen transport to catalytic sites. Generally, we develop the first model of acoustic streaming in unsaturated porous structures.

As shown in Fig. 1, the cathode compartment is the layered structure of three porous materials with different morphology. The role of morphology in wave propagation is crucial and

affects all model parameters, such as Biot parameters and Leverett-function, in other words, water transfer depends on morphology and is different in bi-layer and substrate. At low saturation flow is always channeled and water can be decomposed into connected water and trapped water. Trapped water does not “feel” external pressure and, subsequently does not move, blocking oxygen paths. Trapped water is the “trace” of water “lost from the mainstream.” There are two competitive processes – imbibitions (from main stream) and drainage to the mainstream and structural or acoustic waves depending on their parameters might effectively “clean” or vise versa “spurge” the porous media changing the effective cross section of oxygen path. Only structural vibrations might induce transport of such trapped water “mega-droplets”. Parameters and characteristics of the most beneficial waves (for example, rotational or compression) need to be defined by numerical analysis for real-life geometry using the implemented model.

The plan of this paper is as follows: 1) Problem formulation (above). 2) It is continued with physics-based model approach and equations’ derivation. 3) Testing technique and results we conducted in order to calibrate and verify the model predictions. 4) Results of numerical analysis and 5) we close this paper with some concluding remarks.

## 2 Physical phenomena included in the model.

- The model separates elastic and plastic properties of three immiscible phases involved in mass and momentum transfer on the basis of difference in time scale for vibration (elastic part) and filtration (plastic part). The plasticity of the solid porous phase is neglected as well as all temperature effects.
- The vibration part is represented as three interconnected elastic bodies, of which only solid having non-zero shear modulus. Two types of dissipative mechanisms have been included: damping of the vibration proportional to velocity gradients and momentum transfer due to phase-to-phase interaction.
- The filtration part is modeled as the motion in slowly varying static pressure fields, dissipation mechanism being of D’Arcy type i.e. proportional to relative velocities of liquid and gas.
- The above-mentioned two types of motion interact through dependency of saturation field at every point. It means that elastic parameters and slowly varying static pressures are functions of saturation. Two mass transfer equations for liquid and gas phases contain contributions from both vibration and filtration parts.
- A constitutive relation for two static pressures and the rate of saturation change closes the system. This equation determines the stationary state and, therefore, remaining liquid after all transient process dies out.

Of special interest are non-classical boundary conditions that result in the net de-saturation of the porous sample.

## 3. Scale for description of multiphase flow in vibrating porous medium.

For the description of the flow in the real macro structures one requires the scale of description  $l$  that is much less than the

structure size but much larger than the size of a pore of the porous material -  $\{l_{pore} \ll l \ll L_{device}\}$ . This allows for continuum mechanics type of description of fluid flow and vibration wave propagation, with the structure specific being taken care of by the geometry and boundary conditions of the domain and with porous medium specifics being incorporated into coefficients of the resulting system of equations. The later must be either measured experimentally or calculated with other methods that allow explicit description of the material morphology.

The usual procedure (see e.g. [4 and 5]) is to split the solution domain into sub-domains of the size  $l$ , and substitute all pertinent quantities like kinetic and elastic energies, mass flows, stresses and local pressures, with the quantities averaged of the volume of the sub-domain. A [minimal] statistically representative material volume is called Representative Volume Element (RVE) [5]. An RVE must include a large number of micro-elements to represent local continuum properties. Thus, for Toray paper (sub-layer) with the typical pore-element size is about  $20\mu$  width and  $3\mu$  depth, average size of RVE will be a block with the dimensions  $l \times l \times b \sim 100\mu \times 100\mu \times 20\mu$ . For the by-layer with the typical size of  $3\mu$  the average RVE size is  $15\mu$  to ensure that at least a hundred micro structural objects are averaged in the RVE (as can be seen from the micrographs in Fig 1.). In further consideration all sub-volumes are considered as material points and all differential operations are understood in space of these points.

#### 4. Lagrange formulation of the elastic vibration and dissipation

The complete problem formulation of elastic waves propagation in porous medium filled with several fluids requires formulation of conservative part that results in vibration of all constitutive media and dissipative part that accounts for fluid viscosity and elastic energy dissipation in the solid matrix. For correct account of vibration impact on the flow dissipative and elastic terms must be mixed in visco-elastic constitutive relations of Maxwell-Voight type that involve both stress-strain and stress-strain rates pairs. Although for many materials parameters of visco-elastics constitutive relations have been measured and/or computed, the same for porous materials and especially for the recently invented porous membranes are absent. Here, we suggest simplified approach that is based on the fact that the time scales of vibration induced with the external source in kHz range and higher and filtration of fluids in the porous matrixes are very well separated. Additionally, computational implementation becomes much more simple as it allows natural computation of elastic part in Lagrangian coordinates and filtration part in the Eulerian ones.

The problem, therefore, is to split the formulation of Free Energy functional into Lagrange function for elastic vibration of all constituents, which has originally been formulated by Biot [2], and the part that is responsible for filtration due to slow varying gradients of hydrostatic pressures. Subsequently, various dissipation mechanisms are included to obtain generalization of the Darcy law.

Thus, the goal is to derive expressions for kinetic energy  $K$ , elastic energy  $U$  and dissipative function  $R$  in terms of averaged over an RVE physical quantities. Having in mind that generally all three functions depend on  $\{\mathbf{u}, \mathbf{v}, \partial\mathbf{u}/\partial x\}$ , the equations have the following form:

$$\frac{\partial}{\partial x} \left( \frac{\partial L}{\partial(\partial\mathbf{u}/\partial x)} \right) + \frac{\partial}{\partial x} \left( \frac{\partial R}{\partial(\partial\mathbf{v}/\partial x)} \right) + \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial\mathbf{v}} \right) - \frac{\partial L}{\partial\mathbf{u}} - \frac{\partial R}{\partial\mathbf{v}} = 0 \quad (1)$$

where  $L = K - U$  and  $x$  is coordinate vector.

Theory of liquid flow through porous media uses equation of mass balance closed with the help of Darcy equation. It is the purpose of this work to show how vibration waves described by the system (1) influence filtration rate. The main philosophy of this approach is to add small elastic vibrations and mass transport induced by them "on top" of viscous flow. It is done in the further paragraphs.

#### 4.1 Kinetic energy $K$ .

Original Biot equations [2] have been derived for only one phase filling the porous medium. Two or more phases coexisting inside porous matrix require additional treatment and general guidelines on how to write correctly the mass tensor [6]. We generalize this approach for the multiphase situation in order to obtain consistent expression for the kinetic energy and inertia force as functions of constituents. We are looking for expression for kinetic energy  $K = \frac{1}{2}(\rho_s \mathbf{v}_s^2 + \rho_l \mathbf{v}_l^2 + \rho_g \mathbf{v}_g^2)$

through averaged over some small volume of porous medium  $V$  values of  $\{\langle \mathbf{v}_s \rangle, \langle \mathbf{v}_l \rangle, \langle \mathbf{v}_g \rangle\}$ , where symbol  $\langle \dots \rangle = \frac{1}{V} \int_V dV \langle \dots \rangle$  means averaging procedure over a RVE.

Each phase velocity  $\{\mathbf{v}_s, \mathbf{v}_l, \mathbf{v}_g\}$  can be re-written as a sum of its average value and a deviation as follows:  $\{\langle \mathbf{v}_s \rangle + \Delta\mathbf{v}_s, \langle \mathbf{v}_l \rangle + \Delta\mathbf{v}_l, \langle \mathbf{v}_g \rangle + \Delta\mathbf{v}_g\}$ . Spatially averaged kinetic energy is a sum of terms with averaged squares of partial velocities, which can be expressed in the following form:

$$\langle K \rangle = \frac{1}{2} \left( \rho_s \varepsilon_s \langle (\langle \mathbf{v}_s \rangle + \Delta\mathbf{v}_s)^2 \rangle + \rho_l \varepsilon_l \langle (\langle \mathbf{v}_l \rangle + \Delta\mathbf{v}_l)^2 \rangle + \rho_g \varepsilon_g \langle (\langle \mathbf{v}_g \rangle + \Delta\mathbf{v}_g)^2 \rangle \right)$$

It is important to note that volume fractions  $\{\varepsilon_s, \varepsilon_l, \varepsilon_g\}$  appear before averaging operation. Modifying formula above we get the expression for the averaged over RVE kinetic energy:

$$\langle K \rangle = \frac{1}{2} \left( \rho_s \varepsilon_s \langle \mathbf{v}_s^2 \rangle + \rho_l \varepsilon_l \langle \mathbf{v}_l^2 \rangle + \rho_g \varepsilon_g \langle \mathbf{v}_g^2 \rangle + \rho_s \varepsilon_s \langle (\Delta\mathbf{v}_s)^2 \rangle + \rho_l \varepsilon_l \langle (\Delta\mathbf{v}_l)^2 \rangle + \rho_g \varepsilon_g \langle (\Delta\mathbf{v}_g)^2 \rangle \right) \quad (2)$$

Due to solid matrix rigidity the term  $\rho_s \varepsilon_s \langle (\Delta\mathbf{v}_s)^2 \rangle$  is usually small and we neglect it from further consideration. To close system of equations of motion one has to assume relations

between  $\left\langle (\Delta \mathbf{v}_l)^2 \right\rangle, \left\langle (\Delta \mathbf{v}_g)^2 \right\rangle$  and  $\left\langle \mathbf{v}_s \right\rangle, \left\langle \mathbf{v}_l \right\rangle, \left\langle \mathbf{v}_g \right\rangle$ . To obtain classical Biot kinetic energy terms for two phases flow, only  $\left\langle (\Delta \mathbf{w}_l)^2 \right\rangle, \left\langle (\Delta \mathbf{w}_g)^2 \right\rangle$  should be expressed through mean velocities of constituents:

$$\left\langle (\Delta \mathbf{v}_l)^2 \right\rangle = \begin{vmatrix} \beta_l & \beta_{gl} \\ \beta_{gl} & \beta_g \end{vmatrix} \cdot \begin{vmatrix} \langle \mathbf{v}_l - \mathbf{v}_s \rangle^2 \\ \langle \mathbf{v}_g - \mathbf{v}_s \rangle^2 \end{vmatrix} \quad (3)$$

This parameterization is similar to the assumption of Poissonian statistics for velocities fluctuations. The final expression for the average kinetic energy in this case is as follows:

$$2\langle K \rangle = \rho_s \varepsilon_s \langle \mathbf{v}_s \rangle^2 + \rho_l \varepsilon_l \langle \mathbf{v}_l \rangle^2 + \rho_g \varepsilon_g \langle \mathbf{v}_g \rangle^2 + (\rho_l \varepsilon_l \beta_l + \rho_g \varepsilon_g \beta_{gl}) (\langle \mathbf{v}_l \rangle - \langle \mathbf{v}_s \rangle)^2 + (\rho_l \varepsilon_l \beta_{gl} + \rho_g \varepsilon_g \beta_g) (\langle \mathbf{v}_g \rangle - \langle \mathbf{v}_s \rangle)^2 \quad (4)$$

One phase situation that has been treated by Biot corresponds to only one term  $-\langle (\Delta \mathbf{v}_l)^2 \rangle$  and  $2\beta_l + 1 = a$  - tortuosity in Biot's notation. In the two-fluid model  $\beta_{gl}$  describes correlation between relative motion of gas versus solid and correspondent liquid motion and vice versa.

The dynamic force acting on a system is the derivative of the kinetic energy with respect to velocity. Using (4) we can obtain the following form for (inertia) D'Alamber force:

$$\mathbf{F}^{D'Alamber} = \begin{pmatrix} \mathbf{F}_s^{D'Alamber} \\ \mathbf{F}_l^{D'Alamber} \\ \mathbf{F}_g^{D'Alamber} \end{pmatrix} = \frac{\partial}{\partial t} \cdot \begin{pmatrix} \frac{\partial}{\partial \mathbf{v}_s} \langle K \rangle \\ \frac{\partial}{\partial \mathbf{v}_l} \langle K \rangle \\ \frac{\partial}{\partial \mathbf{v}_g} \langle K \rangle \end{pmatrix} = \frac{\partial}{\partial t} \cdot \begin{pmatrix} \rho_s \varepsilon_s + \rho_l \varepsilon_l \beta_l + \rho_g \varepsilon_g \beta_{gl} & -\rho_l \varepsilon_l \beta_l - \rho_g \varepsilon_g \beta_{gl} & -\rho_l \varepsilon_l \beta_{gl} - \rho_g \varepsilon_g \beta_g \\ \rho_l \varepsilon_l \beta_{gl} + \rho_g \varepsilon_g \beta_g & \rho_l \varepsilon_l + \rho_l \varepsilon_l \beta_l + \rho_g \varepsilon_g \beta_{gl} & 0 \\ -\rho_l \varepsilon_l \beta_{gl} - \rho_g \varepsilon_g \beta_g & 0 & \rho_g \varepsilon_g + \rho_l \varepsilon_l \beta_{gl} + \rho_g \varepsilon_g \beta_g \end{pmatrix} \cdot \begin{pmatrix} \mathbf{v}_s \\ \mathbf{v}_l \\ \mathbf{v}_g \end{pmatrix} \quad (4a)$$

It is important that all elements of mass matrix explicitly depend on volume fractions of constituents, that easy to relate with local fluid/gas saturations. This is the new result because classical Biot theory considers only fully saturated porous media and the solid body porosity is the primary characteristics of that. Now, the mass matrix depends on volume fractions and cross-correlations between relative phases motions that need to be approximated and/or experimentally measured.

## 4.2 Elastic energy U

Usage of averaging procedure of the previous paragraph allows explaining non-diagonal form of the mass tensor in the Biot model [2]. It also provides connection between experimentally measured fluid densities and effective densities that is to be

used in equation for wave propagation (1). One can also try to use the same averaging procedure as was used above, for the derivation of the averaged elastic energy  $U$  as a function of displacements averaged over RVE. This would be analogous to derivation of turbulent models, fluctuations being imposed by the randomness of the porous matrix rather than by the local flow instability. However, those expressions strongly depend on material morphology [4]. Here, we follow more pragmatic approach of Biot [2] and derive expressions<sup>1</sup> for elastic energy with parameters being measurable in specified set of experiments or calculated using direct microscale modeling. With standard notations of theory of elasticity small strain is

$$e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \text{ and } \theta = e_{11} + e_{22} + e_{33} \text{ is dilatation.}$$

Elastic energy is a quadratic expression with respect to elastic strains, which can be written in the general form as follows:

$$U = \frac{1}{2} \lambda \cdot \theta^2 + 2\mu \cdot \sum_{ij} e_{ij} \cdot e_{ij} + \mathfrak{I}_l(\theta_s, \theta_l) + \mathfrak{I}_g(\theta_s, \theta_g) \quad (5)$$

Where two first terms represent self elastic energy of the porous solid and two last terms are energy of liquid and gas dependent on solid distortion.

We assume there is no shear in gas and liquid phases, taking into account that liquid flow rate in porous media is very low. Therefore, the elastic energy for gas and liquid phases depends only on strain dilatation. Stress tensor for solid matrix and pressures for liquid and gas can be obtained by taken correspondent derivatives:

$$\begin{aligned} \sigma_{ij}^s &= \frac{\partial U}{\partial e_{ij}} = \lambda \cdot \theta_s \cdot \delta_{ij} + 2 \cdot \mu \cdot e_{ij} + \frac{\partial \mathfrak{I}_l(\theta_s, \theta_l)}{\partial \theta_s} + \frac{\partial \mathfrak{I}_g(\theta_s, \theta_g)}{\partial \theta_s} \\ \sigma_{ij}^l &= -\varepsilon_l \cdot p_l \cdot \delta_{ij} = \frac{\partial U}{\partial \theta_l} = \frac{\partial \mathfrak{I}_l(\theta_s, \theta_l)}{\partial \theta_l} \\ \sigma_{ij}^g &= -\varepsilon_g \cdot p_g \cdot \delta_{ij} = \frac{\partial U}{\partial \theta_g} = \frac{\partial \mathfrak{I}_g(\theta_s, \theta_g)}{\partial \theta_g} \end{aligned} \quad (6)$$

Here  $\delta_{ij}$  is Kronecker delta. Functions  $\mathfrak{I}_{l,g}(\theta_s, \theta_{l,g})$  account for the pressure change due to liquid and gas motion under solid deformation. Below we consider some special cases and the corresponding potential energy forms.

In the absence of large motions and near equilibrium between all three phases,  $U$  can be expanded up to the second terms in  $(\theta_s, \theta_{l,g})$ . We emphasize here that the theory we are constructing in this work is an elastic correction to the "pre-existing" viscous flow. This assumption is fully justified for small vibrations, when signal magnitude does not exceed 5% of structural characteristic size. Naturally, bigger loads would lead to the structure fatigue failure.<sup>2</sup>

<sup>1</sup> Of course, we use phenomenological considerations here, so it is derivation in physical but pure mathematical sense.

<sup>2</sup> (Here,  $(\overset{filtr}{\Theta}_g, \overset{filtr}{\Theta}_l)$  are quasi equilibrium values of liquid and gas dilations caused by static pressures, conditions of quasi equilibrium being  $\frac{\partial \mathfrak{I}_{l,g}(\theta_s, \overset{filtr}{\Theta}_{l,g})}{\partial \theta_{l,g}} = 0$ .)

Total elastic energy is the following sum:

$$U = U_s + U_{sl} + U_{sg} + U_l + U_g + U_{gl},$$

where

$$U_s = \frac{1}{2} \lambda \cdot \theta_s^2 + 2 \cdot \mu \cdot \sum_{ij} e_{ij} \cdot e_{ij} \text{ self-elastic energy of porous}$$

solid;  $U_{sl} = M_{ls} \cdot \theta_l \cdot \theta_s + M_{gs} \cdot \theta_g \cdot \theta_s$  - energy of interaction

between liquid and solid and gas and solid;  $U_l = \frac{1}{2} M_l \cdot \theta_l^2$ ;

and  $U_g = \frac{1}{2} M_g \cdot \theta_g^2$  -self-elastic energy of liquid and gas;

$M_{gl} \cdot \theta_l \cdot \theta_g$  -energy of interaction between liquid and gas;

$P_l \cdot \Theta_l + P_g \cdot \Theta_g$  -energy of interaction between liquid and gas

and correspondent static pressures; and simple form for

$\mathfrak{S}_{l,g}(\theta_s, \Theta_{l,g}) = P_{l,g} \cdot \varepsilon_{l,g} \cdot \Theta_{l,g}$  has been chosen. Here,  $M_l$

and  $M_g$  are self influence bulk coefficients for liquid and gas

correspondently,  $M_i$ , ( $i = ls, gs, gl$ ) are coefficients of mutual interaction. Constitutive relations relating stresses and strains are obtained by differentiation of elastic energy with respect to strain tensor and has the form:

$$\begin{aligned} \sigma_{ij}^s &= \frac{\partial U}{\partial e_{ij}} = \lambda \cdot \theta_s \cdot \delta_{ij} + 2 \cdot \mu \cdot e_{ij} + M_{ls} \cdot \theta_l + M_{gs} \cdot \theta_g \\ \sigma_{ij}^l &= -\varepsilon_l \cdot p_l \cdot \delta_{ij} = \frac{\partial U}{\partial \theta_l} = M_l \cdot \theta_l + M_{ls} \cdot \theta_s + M_{gl} \cdot \theta_g \quad (7) \\ \sigma_{ij}^g &= -\varepsilon_g \cdot p_g \cdot \delta_{ij} = \frac{\partial U}{\partial \theta_g} = M_g \cdot \theta_g + M_{gs} \cdot \theta_s + M_{gl} \cdot \theta_l \end{aligned}$$

### 4.3 Biot theory

In his original paper [2] Biot considered fully saturated porous media and disregarded porous pressure  $P_l^{pore}$ . Under these simplifications, from relationship (7) setting all gas-related parameters as well as  $P_l^{pore}$  to zero we obtain the constitutive relations:

$$\begin{aligned} \sigma_{ij}^s &= \frac{\partial U}{\partial e_{ij}} = \lambda \cdot \theta_s \cdot \delta_{ij} + 2 \cdot \mu \cdot e_{ij} + M_l \cdot \gamma_l \cdot \theta_l \\ \sigma_{ij}^l &= -\phi \cdot p_l \cdot \delta_{ij} = \frac{\partial U}{\partial \theta_l} = M_l \cdot \theta_l + M_l \cdot \gamma_l \cdot \theta_s \end{aligned} \quad (8)$$

We substituted liquid volume fraction from (7) with material porosity. Due to the fact that solid is fully saturated; liquid volume fraction is identically equal to the porosity. Thus, we deduced Biot's constitutive model from ours as a limiting case.

## 5. Dissipative function R

Dissipative function determines amount of energy that mechanical system losses per unit time [7]. Dissipative processes are always irreversible, so the dissipative function

should not change sign and, consequently, to the first approximation should be positive definite quadratic form. We assume there are three major dissipative processes in the porous partially saturated continuum. First, it is energy losses associated with the friction due to liquid/gas phase relative flow with respect to the solid matrix. We would call this D'Arcy term  $R^{D'Arcy}$ , emphasizing the mass-transfer role in this process. The second, energy is lost due to waves damping/attenuation in the solid matrix, or in other words,  $R^{Solid}$  terms responsible for viscous friction in dry solid matrix. The third term  $\mathfrak{R}_{gl}$  is the energy dissipation rate due to spatial gradients of relative solid-liquid and solid-gas motion. This term, in particularly, describes hysteretic effects.

$$R = R^{D'Arcy} + R^{Solid} + \mathfrak{R}_{gl} = \quad (9)$$

$$\begin{aligned} &\frac{\mu_l}{2 \cdot K_0} \langle \mathbf{w}_l \rangle^T \mathbf{K}_l^{-1} \langle \mathbf{w}_l \rangle + \frac{\mu_g}{2 \cdot K_0} \langle \mathbf{w}_g \rangle^T \mathbf{K}_g^{-1} \langle \mathbf{w}_g \rangle + \\ &\frac{\sqrt{\mu_l \cdot \mu_g}}{K_0} \langle \mathbf{w}_g \rangle^T \mathbf{K}_{gl}^{-1} \langle \mathbf{w}_l \rangle + \frac{1}{2} \zeta \cdot \langle \mathcal{G}_s \rangle^2 + 2 \cdot \eta \cdot \sum_{ij} v_{ij}^s v_{ij}^s + \mathfrak{R}_{gl}(\omega_l, \omega_g, \mathcal{G}_s) \end{aligned}$$

Here we denote throughout vectors  $\mathbf{w}_{l,g} = \varepsilon_{l,g} \cdot (\mathbf{v}_{l,g} - \mathbf{v}_s)$  filtration velocities of liquid and gas. We also introduce symmetric part of velocity gradient tensor  $v_{ij}^s$  for solid matrix – strain rate tensor and solid strain dilatation rate -  $\mathcal{G}_s$  as well as filtration dilatation rate for liquid and gas as follows:

$$v_{ij}^s = \frac{1}{2} \left( \frac{\partial v_i^s}{\partial x_j} + \frac{\partial v_j^s}{\partial x_i} \right), \quad \mathcal{G}_s = v_{11}^s + v_{22}^s + v_{33}^s, \quad \omega_{l,g} = \frac{\partial w_{l,g}^1}{\partial x_1} + \frac{\partial w_{l,g}^2}{\partial x_2} + \frac{\partial w_{l,g}^3}{\partial x_3}$$

Dissipation function  $R^{Solid}$  describes inner friction and should be zero if there is no internal motion [7], particularly in case of rigid motion. It means that this term should depend on velocities gradients. For isotropic body two viscosities (bulk and shear) should be introduced as it shown in Eq. (9).

In special case of weak liquid-gas interaction  $\mathfrak{R}_{gl}(\omega_l, \omega_g, \mathcal{G}_s)$  can be simplified to just a sum  $\mathfrak{R}_l(\omega_l) + \mathfrak{R}_g(\omega_g)$ . Under assumption of small oscillations near equilibrium,  $\mathfrak{R}_{gl}(\omega_l, \omega_g, \mathcal{G}_s)$  has a general quadratic form [8]:

$$\begin{aligned} \mathfrak{R}_{gl}(\omega_l, \omega_g, \mathcal{G}_s) &= \frac{1}{2} R_l \cdot \omega_l^2 + \frac{1}{2} R_g \cdot \omega_g^2 + R_{gl} \cdot \omega_l \cdot \omega_g + \\ &R_{ls} \cdot \omega_l \cdot \mathcal{G}_s + R_{gs} \cdot \omega_g \cdot \mathcal{G}_s \end{aligned}$$

In this case, the corresponding friction forces are linear with filtration dilatations rates and equal to zero at zero dilatation rates. To include hysteresis into this formalism, non-quadratic form of  $\mathfrak{R}_{l,g}(\omega_{l,g})$  must be preserved.

According to the basic equation (1) the corresponding dissipative forces can be obtained from the dissipative function by derivation with respect to appropriate velocities and velocities gradients. We start with forces caused by filtration.

The first three terms in the right hand site of (9) give the general expression for Darcy type forces:

$$\begin{aligned}\mathbf{F}_s^{D'Arcy} &= \frac{\partial R}{\partial \mathbf{v}_s} = -\frac{\partial R}{\partial \mathbf{v}_l} - \frac{\partial R}{\partial \mathbf{v}_g} \\ \mathbf{F}_l^{D'Arcy} &= \frac{\partial R}{\partial \mathbf{v}_l} = \varepsilon_l \frac{\mu_l}{K_0} \mathbf{K}_l^{-1} \mathbf{w}_l + \varepsilon_l \frac{\sqrt{\mu_l \cdot \mu_g}}{K_0} \mathbf{K}_{gl}^{-1} \mathbf{w}_g \quad (10) \\ \mathbf{F}_g^{D'Arcy} &= \frac{\partial R}{\partial \mathbf{v}_g} = \varepsilon_g \frac{\mu_g}{K_0} \mathbf{K}_g^{-1} \mathbf{w}_g + \varepsilon_g \frac{\sqrt{\mu_l \cdot \mu_g}}{K_0} \mathbf{K}_{gl}^{-1} \mathbf{w}_l\end{aligned}$$

As everywhere in this paper, subscripts (s, l, and g) correspond to solid, liquid, and gas phases accordingly.  $\mathbf{K}_l$ ,  $\mathbf{K}_g$ , and  $\mathbf{K}_{gl}$  are tensors for liquid permeability, gas permeability and cross liquid-gas permeability. Last term characterizes the flux of one of species when pressure is applied to another. All permeability tensors components are functions of liquid saturation  $S = \varepsilon_l / \phi$  or  $\varepsilon_l$  and  $\varepsilon_g$  accordingly.

The velocities gradients dependent terms contribute to internal stresses and need to be added to the generic expressions for completion of constitutive relations:

$$\begin{aligned}\sigma_{ij}^{s\ diss} &= \frac{\partial R^{Solid}}{\partial v_{ij}} + \frac{\partial \mathfrak{R}_{gl}}{\partial v_{ij}} \\ \zeta \cdot \mathcal{G}_s \cdot \delta_{ij} + 2 \cdot \eta \cdot v_{ij} - \varepsilon_l \frac{\partial \mathfrak{R}_l(\omega_l)}{\partial \omega_l} \delta_{ij} - \varepsilon_g \frac{\partial \mathfrak{R}_g(\omega_g)}{\partial \omega_g} \delta_{ij} \quad (11)\end{aligned}$$

$$\begin{aligned}\sigma_{ij}^{l\ diss} &= -\varepsilon_l \cdot p_l^{diss} \cdot \delta_{ij} = \frac{\partial \mathfrak{R}_l}{\partial v_l} = \varepsilon_l \frac{\partial \mathfrak{R}_l(\omega_l)}{\partial \omega_l} \delta_{ij} \\ \sigma_{ij}^{g\ diss} &= -\varepsilon_g \cdot p_g^{diss} \cdot \delta_{ij} = \frac{\partial \mathfrak{R}_g}{\partial v_g} = \varepsilon_g \frac{\partial \mathfrak{R}_g(\omega_g)}{\partial \omega_g} \delta_{ij}\end{aligned}$$

Also, relaxation forces for filtration of gas and liquid are:

$$\begin{aligned}^{filtr} \mathbf{F}_l^{D'Arcy} &= \frac{\partial R}{\partial \mathbf{V}_l} = (\varepsilon_l)^2 \frac{\mu_l}{K_0} \mathbf{K}_l^{-1} \mathbf{V}_l + (\varepsilon_l)^2 \frac{\sqrt{\mu_l \cdot \mu_g}}{K_0} \mathbf{K}_{gl}^{-1} \mathbf{V}_g \quad (12) \\ ^{filtr} \mathbf{F}_g^{D'Arcy} &= \frac{\partial R}{\partial \mathbf{V}_g} = (\varepsilon_g)^2 \frac{\mu_g}{K_0} \mathbf{K}_g^{-1} \mathbf{V}_g + (\varepsilon_g)^2 \frac{\sqrt{\mu_l \cdot \mu_g}}{K_0} \mathbf{K}_{gl}^{-1} \mathbf{V}_l\end{aligned}$$

Here, absolute velocities of liquid and gas  $\mathbf{V}_{l,g}$  stand instead of relative velocities  $\mathbf{w}_{l,g} = \varepsilon_{l,g} \cdot (\mathbf{v}_{l,g} - \mathbf{v}_s)$  as filtration velocity of solid is zero - non-consolidating porous material.

## 6 J-Leverett function

To provide a model of two non-mixing fluids that includes effect of residual saturation we have to account for strong dependence of material properties on saturation and wettability.

Liquid saturation is defined as  $S = \frac{\varepsilon_l}{\phi} = \frac{\varepsilon_l}{1 - \varepsilon_s}$ . Capillary

pressure  $P_c$  is usually defined as the pressure difference between two (wetting and non-wetting) phases as a function of the saturation. For modeling and correlation purposes the capillary pressure can be described by a dimensionless

Leverett-J function:  $J = \frac{\Delta P(S)}{\sigma} \sqrt{\frac{K}{\phi}}$ , where  $\sigma$  is interfacial

tension between the two phases and  $\phi$  is the porosity of the solid matrix. Definitely, we can write down the same kind of functions for each phase and their difference would determine the capillary pressure. Leverett function correlates data over several orders of magnitude change in permeability as stated in [12]. Capillary pressure is a measure of the curvature of the interface separating phases, therefore it is a strong function of morphology. During a subsequent water-drive and turns off some amount of water eventually remains capillary trapped, floating as disconnected blobs in the pores. The residual water saturation  $S_0$  is determined by the pore space morphology and the structure wettability.  $S_0$  is called the ‘‘irreducible saturation’’, which correlates with the amount of trapped water and appears to be around 0.05. Simplified form of the J

function is:  $J(S) = -\frac{C_0(S - S_{middle})}{(S - S_0)(1 - S)}$ , where  $S_{middle}$  is the

dynamic saturation equilibrium specific for the interacting materials..

We have conducted tests intended to measure liquid water permeability of the cathode layers under vibration, static pressure and their combination. Under operating conditions, water is wicked away by the pressure difference between catalytic layer (where water is generated) and water transport plate (WTP) with negative pressure (sink). By calibrating model predictions against experimental measurements we deduced the value of  $C_0 = 0.9e5$ .

## 7. Equations of filtration for two non-mixing fluids and generalized D'Arcy law.

Fluid filtration in porous media is traditionally described by equations based on mass conservation [4, 9, - 12] rather than on Biot type equation [2]:

$$\begin{aligned}\frac{\partial}{\partial t}(\varepsilon_l \rho_l) + \nabla \cdot (\rho_l (\mathbf{V}_l + \mathbf{w}_l)) &= 0 \\ \frac{\partial}{\partial t}(\varepsilon_g \rho_g) + \nabla \cdot (\rho_g (\mathbf{V}_g + \mathbf{w}_g)) &= 0\end{aligned} \quad \mathbf{w}_{l,g} = \varepsilon_{l,g} \cdot (\mathbf{v}_{l,g} - \mathbf{v}_s) \quad (13)$$

In the case of *saturated incompressible fluids* those equations are simply direct consequences of the definition of the dilatation:  $\theta_l \equiv \nabla \cdot (\mathbf{u}_l - \mathbf{u}_s)$ . In the two-fluid case when

saturation  $\varepsilon_l$  and  $\varepsilon_g$  change both in time and space, situation becomes more complicated especially taking in mind that Leverett functions and dissipation are measured as functions of  $\varepsilon_l$  and  $\varepsilon_g$ . One can consider, therefore, the two continuity equations as equations for additional variable  $\varepsilon_{l,g}$  and solve them together with Biot equations for vibration. Note that

because the continuity equations are conservation laws and expressed as full time derivatives, their contribution to Lagrange equations taken with the Lagrange multipliers don't change equations of motion. Equilibrium condition requires that the resultant of all applied forces be compensated by internal stresses. For the liquid the total pressure gradient should be equal to the sum of D'Alamber and D'Arcy forces

$$\nabla(-\varepsilon_l p_l) = \mathbf{F}_l^{\text{D'Alamber}} + \mathbf{F}_l^{\text{D'Arcy}} \quad (14)$$

Next, without loss of generality, we neglect all "gas terms" and obtain the simple form of the generalized D'Arcy law for unsaturated porous media under acoustic loading. Expressions for liquid components of these forces are given in the derived expressions (4a) and (10) accordingly. Thus, eq. (14) can be rewritten in the following form:

$$\nabla(-\varepsilon_l p_l) = \frac{\partial}{\partial t}(\beta \rho_l \varepsilon_l (\mathbf{v}_l - \mathbf{v}_s)) + \frac{\partial}{\partial t}(\rho_l \varepsilon_l \mathbf{v}_l) + \frac{\varepsilon_l \mu_l}{K_0} \mathbf{K}^{-1} \mathbf{w}_l;$$

The D'Alamber term depends on both relative ( $\mathbf{v}_l - \mathbf{v}_s$ ) and absolute liquid velocities. Resolving this equality with respect to the flux  $\mathbf{q} \triangleq \mathbf{w}$ , and also taking into account that  $p_l = P^{\text{static}} - M\theta_l - M\gamma\theta_s$ , we immediately obtain:

$$\mathbf{q} = -\frac{K_0 \mathbf{K}}{\mu_l} \left( \frac{\nabla(\varepsilon_l P)}{\varepsilon_l} - \frac{M}{\varepsilon_l} (\nabla(\varepsilon_l \theta_l) + \gamma \nabla(\varepsilon_l \theta_s)) + \frac{\beta \rho_l}{\varepsilon_l} \frac{\partial}{\partial t} \left( \frac{\partial(\mathbf{u}_l - \mathbf{u}_s)}{\partial t} \right) + \frac{\rho_l}{\varepsilon_l} \frac{\partial}{\partial t} \left( \varepsilon_l \frac{\partial \mathbf{u}_l}{\partial t} \right) \right) \quad (15)$$

Here the flux tensor depends on all components gradient of pressure, dilatations and kinematical terms. This is *the novel generalization of the D'Arcy Law for the acoustically loaded unsaturated porous media*. The first term represents generalization of "traditional" pressure gradient corrected for the variable saturation. Very important, that saturation gradient  $P \frac{\nabla \varepsilon}{\varepsilon} = P \nabla \ln \varepsilon$  is also strong driving force specific only for unsaturated inhomogeneously filled porous structure. This term plays an important role but only if either external pressure or "external" dilatation has been applied. The second term explains the role of solid and liquid deformation as a driving force for the flux. Due to dilatation, fluid is squeezed out of the compressed pores due to induced increased fluid pressure in the "extended" area with reduced liquid pressure. The two last terms show the role of dynamics (acoustics) on the liquid flux.

Using the new formulation (15) and assuming everywhere that saturation does not change significantly, we might estimate orders of magnitude for the required parameters of structural acoustics impact to achieve the comparable effect with static pressure. D'Arcy term for the fuel cell cathode compartment is generated by application of the pressure of 1 *psi* over the distance of 200  $\mu$ . Thus, the "static" pressure gradient is of the order of  $\nabla p \approx \frac{1 \text{ psi}}{200 \mu\text{m}} = 34E6 \frac{\text{Pa}}{\text{m}}$ . Letting magnitude be 1% of

the thickness, we immediately obtain that a dynamic term has the order of magnitude  $\rho_l \frac{\partial^2 u}{\partial t^2} \cong \rho A \omega^2 \approx 1.E-3 \omega^2$  and,

subsequently, equating these two pressures, we obtain that the required frequency should be higher than 50 KHz.

## 8. Constitutive relations and complete system of governing equations

By this point we have described all forces acting in the system and the system possible reactions. We introduced elastic and dissipative models. Combination of "elastic" and "viscous" equations completes physics-based system and allows us to predict the influence of vibration on mass transfer and dynamics of saturation on vibration parameters. As was mentioned already  $J$  is a static part of Leverett function for each component (liquid and gas). The constitutive relations should be supplemented by the definition of the Leverett - capillary function, simplified form of which is:

$$T(\varepsilon_l) \cdot \frac{\partial \varepsilon_l}{\partial t} - P_g + P_l = J(\varepsilon_l) \quad (16)$$

The last equation for  $P_{l,g}$  has been derived in [8] using consideration of Coleman and Noll on positiveness of entropy production. Complete system of governing equations needs to be added to the constitutive relations.

$$\frac{\partial \sigma_{ij}^s}{\partial x_j} + \frac{\partial \sigma_{ij}^{s,diss}}{\partial x_j} = \mathbf{F}_s^{\text{D'Alamber}} + \mathbf{F}_s^{\text{D'Arcy}}$$

$$\nabla(-\varepsilon_l (p_l + p_l^{diss})) = \mathbf{F}_l^{\text{D'Alamber}} + \mathbf{F}_l^{\text{D'Arcy}}$$

$$\nabla(-\varepsilon_g (p_g + p_g^{diss})) = \mathbf{F}_g^{\text{D'Alamber}} + \mathbf{F}_g^{\text{D'Arcy}} \quad (17)$$

$$\nabla(-\varepsilon_l P_l) = \text{filtr} \mathbf{F}_l^{\text{D'Arcy}}$$

$$\nabla(-\varepsilon_g P_g) = \text{filtr} \mathbf{F}_g^{\text{D'Arcy}}$$

The second of these equations we have already used deriving novel generalization of D'Arcy law for unsaturated porous media under impact of dynamic loading. Thus, we have described stress-strain relationship, forces causing these stresses, and constituents displacements and velocities resulting from these stresses. To complete this system description one needs to add the mass transfer equations (filtration).

$$\frac{\partial}{\partial t}(\varepsilon_l \rho_l) + \nabla \cdot (\rho_l (\mathbf{V}_l + \mathbf{w}_l)) = 0 \quad (18)$$

$$\frac{\partial}{\partial t}(\varepsilon_g \rho_g) + \nabla \cdot (\rho_g (\mathbf{V}_g + \mathbf{w}_g)) = 0$$

Finally we have the system of governing equations describing multiphase flow in porous media with acoustic streaming. The transmission of sound through unsaturated porous media can be predicted only in the context of the extended Biot model we have developed here. The water and air move simultaneously with solid matrix.

## 9 Results of Numerical Calculations for Different Boundary Conditions

First we considered the problem in the rectangular with pressure defined as boundary conditions as shown in the

scheme. Vibration is applied to the upper boundary as follows:

$$u_y = A \sin(\omega t - kx), \text{ and } v_y = \frac{\partial u_y}{\partial t}.$$

There are two limiting cases depending on material parameters and BC. The first, when vibration [acoustic pressure] is small with respect to applied to boundaries “static” pressure gradient. The second case corresponds to the situation when acoustic pressure is large in comparison with static pressure gradient.

The results of calculations corresponding to the first situation show that saturation distributions under both static and small vibration conditions are similar to each other and show steep saturation gradient near to sink forming a steady distribution.

The most pronounced effect of vibrations on transport process is substantial acceleration of water transport. Under vibration impact, transient period is insignificant in comparison to pure D’Arcy filtration (static) case. Thus, under pressure boundary conditions and dominating filtration term the water just follows the motion of the solid phase. This leads to fast process stabilization and to small integral effect in flux evaluation as shown in Fig. 2. The homogenization of water distribution is significantly faster under vibration impact. Increase of the

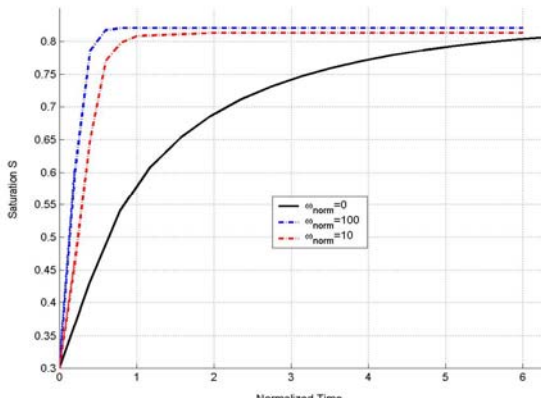


Figure 2. Decrease of transient time with frequency of applied vibration, at normalized model parameters.

effect relates with the increase of the term  $\nabla \cdot (S(v_{water} - v_{solid}))$ . Therefore, this product after averaging should be not only significant but noticeably changes with space to guarantee existence of not small derivative (div). Under this boundary conditions, during each half-cycle water efficiently drains from the bottom boundary and imbibes back during another half cycle. The same situation is observed on the upper boundary. Only with the development of the mechanism preventing back imbibing from the surface or in other words the way of fast removing water from the surface would lead to effective acoustic drying.

Another qualitative situation appears when filtration coefficient is small (as typical for Toray paper and bi-layer) and, therefore, coefficient in front of highest derivative is a

small parameter. This and pressure boundary conditions leads to creation of a boundary layer. The characteristic size of such boundary layer can be evaluated by equating orders of both terms in transport equation above:

$$l \sim KT \sim \frac{k_{water}}{\mu_{water}} T \sim 1 \div 5 \mu m.$$

The thickness of the boundary layer is about several percents of the cathode porous layer. All changes consisting the boundary conditions with the bulk solution take place in this boundary layer (Fig 3) and saturation in the internal part of the porous plate practically does not change with vibrations. As can be seen from the Fig. 3, the region with decreased water content occurs inside the porous plate and the saturation is higher at surface regions, therefore water flux is directed inside the porous plate. In other words, during vibration, plate starts imbibe water, if pressure is defined on the boundary. This is very important situation for PEM FC

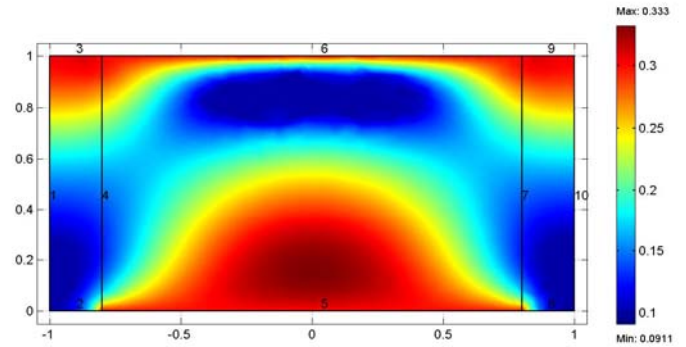


Figure 3. Formation of the boundary layer on the top boundary under acoustic impact. Colors show levels of water saturation.

materials because calculations fit range of material constants. This leads to the conclusion that the most important process of acoustic drying is effective water removal from the surface.

Assuming it is technically possible to realize, we analyze water transfer under this semi-permeable boundary conditions. The modeling results provide us with upper bound estimations of the amount of acoustically induced water flux. Acoustically induced flux is naturally more pronounced if applied static pressure gradient is small. We have numerically analyzed in details the situation when the in-bound water flux at the sink boundary is semi-sealed. We implemented such a semi-permeable physical model by constraining the boundary conditions as  $\int_{boundary} \Gamma \vec{n} dA \geq 0$ . “Low” bottom is semi-

sealed: if  $\int_{boundary} \frac{\partial S}{\partial y} dA > 0$ , then flux is zero, otherwise is not

constrained. Pressure on the “upper” boundary has been set constant  $P=0$ .

Numerical results show that the boundary layer is developed on the boundary where the pressure has been defined. Also, the integrated amount of water in the plate gradually decreases on approximately 5 -10% and stabilizes there. Process starts with relatively fast drainage, then after first half of vibration period the internal pressure gradient is developed in such a way that water would be imbibed from the upper boundary. Up to time  $\sim 4.8$  semi-permeable boundary conditions lead to invariable amount of water content in the porous media (see Fig 4). Then, during another period of vibrations, pressure conditions change

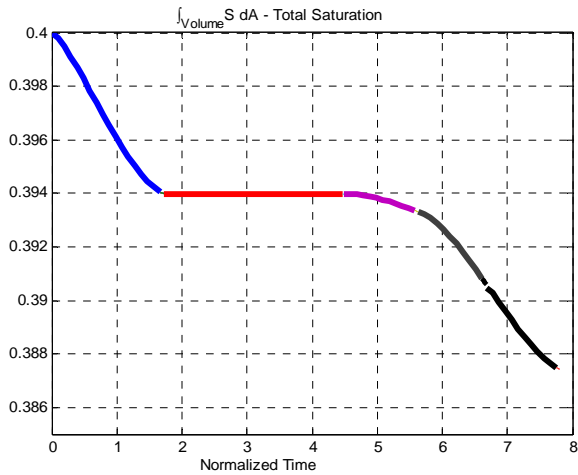


Figure 4. Dynamics of acoustically driven water drainage through semi-permeable boundary

again and drainage takes place. Amount of drained water gradually decreases every period and finally the integrated saturation stabilizes.

The most important conclusion from these calculations can be summarized as follows:

- (i) The acoustically driven water removal from the porous plate is numerically observed;
- (ii) Acoustic drying is possible if and only if water removal from the surface mechanism does exist. Otherwise, water is imbibed back immediately.
- (iii) The acoustic drying/drainage is very fast process,;

## 10. Concluding Remarks

We have developed a unified model of structural/acoustic wave propagation in the PEM FC cathode compartment and coupled it with mass transfer in the porous media. This is the first known to the authors model of such kind coupling acoustics and filtration in a porous media. A set of consistent constitutive relations and governing equations for multiphase partially saturated porous dynamic media have been developed using Lagrangian approach with subsequent inclusion of various dissipative mechanisms. Developed unified methodology is useful for the wide range of Fuel Cell problems as well as for the wide range of other porous structures and could serve as an important design tool.

It has been demonstrated that the phase saturations have strong impact on wave dynamics in porous continuum. Explicit

expressions for generalized multiphase Biot-type coefficients that explicitly depend on the phase saturations have been obtained.

We have succeeded to generalize filtration (D'Arcy) law on dynamic system with water saturation varying with space and time. It has been shown that the vibration-induced flux is a strong function of solid matrix distortion, saturation gradient and inertial forces. We also were able to connect hysteretic phenomena to relaxation process due to gradients of filtration rate.

The developed homogenization technique is crucial for numerical analysis of high frequency impacts. It allows circumventing restrictions on maximum allowed number of finite elements to resolve spatial frequency of the acoustic/structural wave. It has been demonstrated that vibration gives rise to net change of saturation inside porous medium. This effect is analogous to the known phenomenon of acoustic streaming [14 - 16] but has not been discovered before.

We conduct the extremely precise experimental procedure to measure bi-layer/substrate permeability under vibration. Test results for different materials demonstrate (i) existence of the effect, (ii) instability in highly hydrophobic structures under vibration and the strong correlation between induced mass-transport and spatial inhomogeneity generated by structural waves.

The most useful application of the equations derived for time-averaged fluxes is when the leading zero-order approximation is vanished. In the opposite case, vibration-induced fluxes are small and the transport is determined by static pressure gradients. This also specifies potential application of the vibration-induced saturation dynamics. It is most useful when for some reason pressure-induced transport is either impossible or undesirable. Numerical results demonstrate that applied structural vibration/acoustic loading (i) intensifies the process dynamics or in other words decreases time of transient process; (ii) might drain out up to 5-10% of distributed in porous media water if the mechanism preventing reverse acoustic-driven imbibitions is in place; (iii) The fast water removal from the surface is crucial for acoustic-driven drying; (iv) In a working Fuel Cell, vibrations cause increase of the water flux from the porous cathode in both directions out to WTP and back to the catalytic layer, which in turn makes the Fuel Cell performance worse; (v) Acoustic/Structural vibrations accelerates the stabilization of the system (porous media-liquid-gas) at some level predefined by material properties in general and by Leverett function in particular. Therefore, for over-saturated porous media, vibrations lead to fast drying up to stable level and, subsequently, for FC performance improvement. If the initial saturation is smaller then the steady-state level, the porous layer would be immediately imbibed up to the stable level and, subsequently, performance would deteriorates. Both types of phenomena we have observed in practical nature.

On the base of the results mentioned above, we make the following practical conclusions focused on results industrial (fuel cells) implementation: (i) Acoustic/structural vibration drying can be used when other mass-transfer driven processes

are small. It means that the best operational moment for the application of the acoustics is the shutdown drying preventing from possible frosting of the PEM fuel cell cathode compartment. Another optimum performance regime is fuel cell dry start up, when vibrations would significantly cut the start-up stabilization (warming) time. (ii) Fast and efficient water removal from the surfaces and interfaces is crucial for the acoustic drying. Much work needs to be done to develop such a material/technology design. This work shows the direction for the optimal material selection. (iii) Based on numerical results, one can conclude that the acoustically driven material drying (water drainage) is the fast process, therefore, the application of vibration field should be performed as a short impulse series, preferably while source or sink of water is sealed (start-up and shutdown). (iv) Project results show that cathode compartment acoustic cleaning would be beneficial if the cell is turned-off at the moment of cleaning. Therefore subsequent short-time (~1 min) disconnections of one or several cells from the stack for a cleaning would provide positive performance effect.

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