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An Analytical Elucidation of the Influence of Surfactant on Rock Drilling by Shear/Drag Bit

By

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Summary

We explain analytically the effects of surfactants on the enhancement of drilling rates in rock. We propose that the weakening effect is not simply due to the chemical action of the surfactant, as suggested by Rebinder in the 1930's but to a combination of mechanical and chemical interactions. We apply a fracture mechanics approach to elucidate surfactant-assisted drilling. An analysis of the chip formation process during rock cutting by a shear/drag bit has shown that the surfactant-induced damage zone develops in rock primarily in front of the individual bit, and its size can be as much as twice the thickness of the removed chips. This analysis allows us to estimate the influence of surfactants on cutting forces.

1. Introduction

The effects of surfactants have been examined from the standpoint of changes in the surface electrochemistry of rocks. The use of surfactants as drilling fluid additives has been exhaustively studied since Rebinder's works in the 1930–1940's (Rebinder et al., 1948). A number of investigators have confirmed the advantageous effects of chemical additives in drilling fluids, improving the drilling rate by as much as two to three fold in some hard rock types (Westwood and Goldhein, 1970; Engelmann et al., 1987). Paul (1988) pointed out that the formation of double layers of electrically charged particles and a build-up of osmotic pressure in the cracks and pores by as much as 0.1 to 10 atm can promote the fracture process in some rock types.

Lowrison (1974), Fuerstenau et al. (1985) reported that surfactants improve grinding when they are used in the form of grinding aids in wet ball mills. For instance, an anionic additive adsorbs on the solid surface and apparently keeps the fine particles apart through mutual repulsion, thereby reducing the viscosity. Based on lab testing of fracture toughness with various types of chemical additives, Akram and Karfakis (1991) recently suggested that the change of electro-catalytic reactions at the crack tip has an influence on the crack activities during the chip formation process.

Westwood et al. (1974) reported that the optimum drilling rate is achieved when the zeta potential equals zero on the surface of the rock. They also observed the influence of the bit geometry on the drilling rate under the zero potential conditions. Tweeton et al. (1976) conducted drilling tests using marble, serpentine, and quartz in different environments and concluded that there is no obvious correlation between the values of the zeta potential and the drilling rates.

These experimental studies provide substantial information that the effects of surfactants on rock drilling depend not only on chemical compositions but on the type of mechanical load. Yet, the current understanding of the mechanism that causes a change in the drilling rate is only qualitative, at best. Furthermore, the beneficial effect of surfactants in rock drilling has not yet been fully established.

In our model, we hypothesize that the beneficial effects of surfactant-assisted drilling are not due solely to the chemical action of the surfactant, but to a combination of mechanical and chemical interactions. We attempt to verify our hypothesis by analytical means. Sample calculations and experimental evidence supporting the calculations are presented.

2. Analytical Modelling of Bit Penetration Process

In this paper we describe the influence of surfactants on damage zone development and the role this zone plays in decreasing the cutting force. Two systems of fractures, namely, horizontal shear fractures and damage zone cracks (vertical cracks) are created in brittle rock by drilling action (see Fig. 1). The role of these cracks and the degree of their influence on the destructive process are analyzed and estimated.

In full-size bench testing (Staroselsky et al., 1991) on sandstone, we observed a 45%–55% decrease in the cutting force when we used a surfactant as compared to cutting dry rock. In this work we applied the surfactant solution only once, when the first layer of rock was removed, but the cutting force continued to decrease even after the removal of the third layer. This suggests that the size of the damage zone caused by the surfactant is of the order of at least two chip thicknesses. Thus, the usage of the surfactant increases the size of the cracks in the damage zone (Fig. 1). Molecules of the surfactant form diffusion double layers on the crack surfaces. These layers help prevent the crack surfaces from closing (Rebinder and Shchukin, 1973). Under electrostatic forces and forces of molecular interaction, the surfactant molecules move toward crack tips right up to steric hindrance. The force of interaction between the free surfaces of solid and adsorption layers prevents the expulsion of fluid from the crack. The experiment also indicates that the ratio $(Q/K_{\mathrm{Ic}})^2$, which is proportional to the crack length, remains almost constant (where Q is the thrust force applied to the bit and K_{Ic} is the mode I fracture toughness of the rock being cut). The fact that the decrease in resistance to the cutting action is translated to a decrease in applied force explains these experimental results. This fact also suggests that the increase in the damage zone is not attributable to the decrease in K_{Ic} under the influence of surfactant alone.

We consider the following problems: 1. The development of the cracks in the damage zone in front of the cutting tool and behind it, and 2. The interaction of these cracks with shear fractures. We simulate the cutting tool as a rigid bit which moves

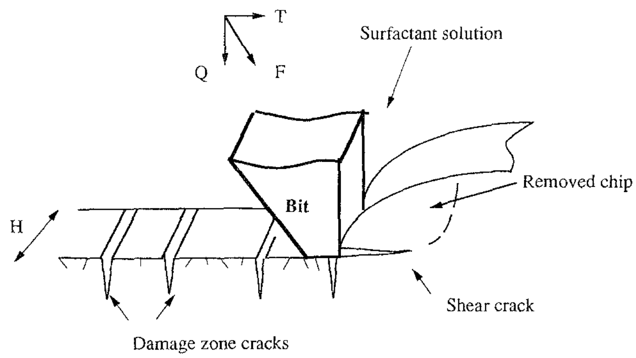

Fig. 1. Scheme of the bit-rock contact. F is applied force, T is cutting force, Q is thrust force, H is the bit width

quasi-statically along the surface of a half plane containing shallow surface cracks. Friction is generated at the bit-rock interface.

We now analyze the first problem listed above. A compression zone is formed in front of the bit. The crack in front of the bit is filled with drilling fluid and is compressed by the load applied by the bit as shown in Fig. 2. The rock stress decreases with increased depth into the half-plane. The fluid pressure is uniformly distributed over the entire fluid-filled crack surface. For this reason the stresses on the sides of the crack near the free surface are compressive, while those near the tip of the crack are tensile. Therefore, the part of the crack outcropping on the free surface (A in Fig. 2) closes to the length d, while the crack tip (B in Fig. 2) propagates up to the length a (B$'$ in Fig. 2). The criterion for the crack propagation is chosen in the form:

$$K_{\rm I} = K_{\rm IC}$$

where $K_{\rm I}$ is mode I stress Intensity Factor (SIF).
In order to find the final crack length a we need to know SIF $K_{\rm I}$ as a function of the length a. Hence, this criterion can be re-written as

$$K_{\rm I}(a) = K_{\rm IC} \tag{1}$$

We determine the SIF from the solution of the elastic problem for the half-plane with an orthogonal edge crack. The boundary conditions on the crack sides for this problem are:

$$\sigma_{\rm n} = \sigma_{\rm x} - p_0, \;\; \tau_{nt} = 0; \;\; d \le y \le b; \tag{2}$$

where

σ_x is the horizontal component of the normal stress in rock induced by bit loading,
p_0 is the fluid pressure in the crack,
σ_n, τ_{nt} are normal and tangential stresses on the crack,
b is length of the initial crack, and
d is length of the closed part of the crack.

The components of the stress tensor, particularly σ_x, have a general form:

$$\sigma = \frac{Q}{lH} g\left(\frac{y}{l}, \rho\right),$$

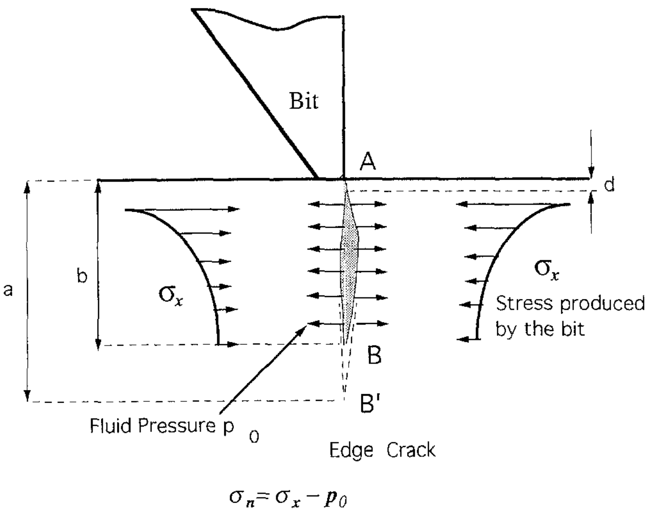

Fig. 2. Schematic drawing of the edge crack filled with surfactant in front of the bit. *b* is initial crack length; *a* is final crack length; *d* is the length of the closed zone; *A* is the point of crack-free surface intersection; *B* is the crack tip before loading; *B'* is final position of the crack tip

where

l is one half of the bit-rock contact length,
H is the width of the bit (Fig. 1),
ρ is a friction coefficient at the bit-rock interface,
Q is a thrust force (Fig. 1),
y is the distance from the free surface of the half plane to the material point where the stress tensor is calculated, and
g is a function known from the solution of the contact elastic problem in a half-plane without cracks (Galin, 1961).

We assume the boundary of the half-plane is stress-free. In formulating this boundary value problem, we substitute for the bit loading the stresses it produces. The crack surfaces are closed to a depth d due to compression.

Cracks in a semi-infinite plate have been examined in the literature. A solution for a crack normal to the semi-plane boundary is given, for example, by Savruk (1981). However, his solution is cumbersome and cannot be used for the determination of fluid pressure p_0 and in criteria of the type (1). The problem (2) can be solved using an asymptotic method which is elaborated in Appendix A. A rough estimate shows that the size of the closed zone d is much less than the crack length a. Thus, we can obtain the asymptotic expression for K_I (d) and K_I (a) under the condition of $d/a << 1$, for which the value of K_I (a) is as follows:

$$\mathrm{K_I}(a) = \sqrt{2ca^{c-1}/\pi^3} \int_0^a \frac{p_0 - \sigma_x}{\sqrt{a^c - t^c}} dt, \tag{3}$$

where $c = \frac{2\pi^2}{\pi^2 - 4}$ and t is a variable of integration.
To find the surfactant fluid pressure p_0 which is used in Eqs. (2) and (3) we solve the

equation describing the closed condition of the crack at point A (see Fig. 2):

$$K_I(d) = 0. \tag{4}$$

Solving this equation produces the result given below:

$$p_0 = \frac{\int_0^a \sigma_x \ln \frac{a^{c/2} + \sqrt{a^c - t^c}}{a^{c/2} - \sqrt{a^c - t^c}} dt}{\int_0^b \ln \frac{a^{c/2} + \sqrt{a^c - t^c}}{a^{c/2} - \sqrt{a^c - t^c}} dt} \tag{5}$$

Here b is the initial and a is the final crack length.
In this solution, the fluid pressure is expressed as a function of crack lengths and the applied stress. Also, we assume that the volume of the fluid filling the crack does not change during the loading action.

The volume of drilling fluid that could be squeezed out of the crack is not expected to exceed 5% of the total volume of the liquid that filled the crack according to an estimate based on Poiseuille's law. Its effect, therefore, is considered negligible. The drilling fluid does not escape easily from the crack since the crack is closed due to bit loading and the high compressional stress field near the exposed surface of the rock. A typical advance rate of the bit and an average duration of the bit loading are estimated to be 0.5 m/sec and 0.005 sec, respectively. The pressure of the drilling fluid in the crack is estimated to be 10^1–10^2 MPa. The crack opening displacement (COD) is of the same order of magnitude as the grain size. The adsorption layers that are generated by the surfactant tend to reduce the crack opening. It should be noted that the viscosity of the surface layer is determined by the surfactant film and is at least 3–5 times higher than the viscosity of water.

Using expressions (3) and (5), we reduce (1) to the algebraic equation related to SIF and the crack length. We are able to do that because σ_x and p_0 are known functions of the crack length.

We find that function $K_I \cdot H \cdot l^{1/2}/Q$ depends on ratio $\frac{a}{b}$, friction coefficient ρ, which is approximately equal to the ratio of cutting force T and thrust force Q (see Fig. 1), and values of normalized fracture toughness $K_{IC} \cdot H \cdot l^{1/2}/Q$. We also suppose that crack extension a–b is approximately equal to the thickness h of removed chips, based on the analysis of the steady-state process. Calculated crack length for test data obtained from the exploratory tests are given below in Table 1. These example calculations show that the damage zone influences the two subsequent bit passes.

Above, we have analyzed crack propagation ahead of the bit where the stress field is predominantly in compression. To complete the analysis of the damage zone development, we need to examine the growth of the cracks behind the bit as well (see Fig. 3a). The crack usually opens under the action of the stresses produced by the bit. In this case the effect of the surfactant is limited to the reduction of the fracture toughness of the rock. To handle this problem we take a different approach from the one we used in the preceding problem. SIF is determined by applying the Green function method. We choose as a Green function the solution for the crack problem in a half-plane when the point load is applied to the boundary (Cartwright and Rooke, 1979). Normal force Q and cutting force T are applied to the half plane, containing the edge

Table 1. Examples of calculations

Parameters	Test 1	Test 2
Rock type	sandstone	sandstone
Bit width	H = 20 mm	H = 5 mm
Bit-rock contact length	l = 2.5 mm	l = 1 mm
Chip thickness	h = 8 mm	h = 5 mm
Thrust force	Q = 7.8 KN	Q = 3.7 KN
Cutting force	T = 2.6 KN	T = 2.1 KN
Fracture toughness under influence of the Surfactant	$K_{Ic} = 0.30\,MPa\sqrt{m}$	$K_{Ic} = 0.30\,MPa\sqrt{m}$
Calculated crack length	a = 18 mm	a = 6 mm

crack of length b. The point of force application is located at a distance l_0 from the crack (Fig. 3b). Then we use approximate solutions (Cartwright and Rooke, 1979) for the stress intensity factors at the crack tip. These expressions have a form:

$$K_I^Q = -\frac{Q}{\sqrt{\pi b}}(1 - l^2)(0.824 + 0.064l - 0.843l^2 + 15.41l^3 - 53.38l^4 + 59.74l^5)$$

$$K_I^T = \frac{T}{\sqrt{\pi b}}(1 - l^2)(1.294 + 0.004l + 0.129l^2 + 0.890l^3 - 22.141l^4 + 10.962l^5)$$

$$l = \frac{l_0}{l_0 + b}; \quad T = -\rho Q. \tag{6}$$

where

b is length of the edge crack;
l_0 is distance between the point of load application and the crack (Fig. 3b);
K_I^Q is the Stress Intensity Factor caused by Q;

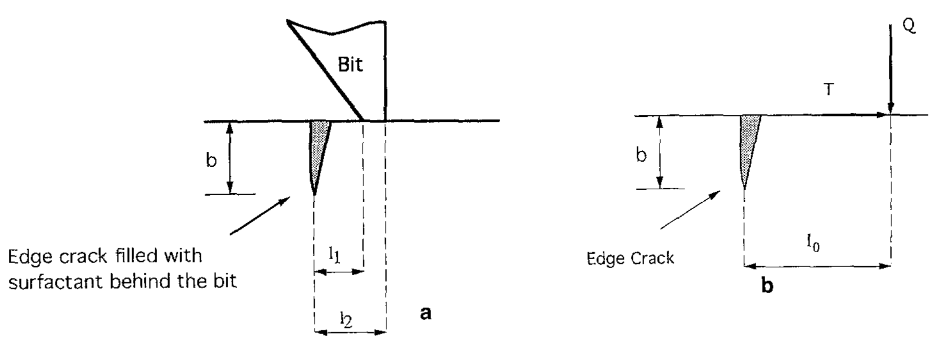

Fig. 3. a. Edge crack filled with surfactant behind the bit. *b.* is initial crack length; l_1 and l_2 are distances from the crack to the front and back sides of the bit respectively. **b.** Edge crack in the field, generated with point force (Cartwirght and Rooke, 1979). Q is a normal (thrust) force acting on the half plane boundary; T is a tangential (cutting) force acting on the half plane boundary; l_0 is distance between the point of load application and the crack

K_I^T is the Stress Intensity Factor caused by T;
ρ is a friction coefficient;
Q is a normal (thrust) force acting on the half plane boundary (Fig. 3b);
T is a tangential (cutting) force acting on the half plane boundary (Fig. 3b).

If a bit acts on the boundary of the half plane (Fig. 3a), the stress intensity factor at the tip of the vertical edge crack is equal to

$$K_I = \int_{l_1}^{l_2} F(x)R(x)dx, \tag{7}$$

where $F(x) = \rho K_I^T(t) - K_I^Q(t),\ t = \frac{x}{x+b},\ x\epsilon(l_1, l_2)$.
The function $R(x)$ is defined by contact stresses (Galin, 1961):

$$R(x) = \frac{\sin \pi\theta}{\pi(l_2 - x)^{1-\theta}(x - l_1)^{\theta}}, \tag{8}$$

where

l_1, l_2 are distances from the near and far sides of the bit to the crack, respectively,
$\theta = \frac{1}{\pi}\arctan\frac{2-2\nu}{\rho(1-2\nu)}$ and ν is the Poisson's ratio.

The calculations show that the maximum value of K_I for a crack behind the bit is less than that for a similar crack in front of the bit. When the bit moves to a distance away, which is one to two times the crack length, the crack opens and remains open for any $p > 0.1$. Because they are open, the cracks in the damage zone behind the bit shield each other, and the influence of bit loading on them decreases rapidly. Thus, cracks located behind the bit do not extend deeper. It is important to point out that these cracks remain open.

So far, we have shown that the damage zone develops around the bit under the influence of surfactants. The average size of the incipient cracks in the damage zone is up to 2.5 times the thickness of the removed chips. These cracks are filled with surfactant and promote ensuing cutting cycles.

The next step is to estimate the influence of the damage zone on the cutting force. We simulate cutting as a repetitive process of brittle chip formation. Our model is based on the Cherepanov et al., (1989) approach for cutting analysis. We consider the process in which the bit acts on the rock with a force T (Fig. 4a). The shear fracture propagates at depth h with the increase of the force T. As soon as T reaches its maximum value, the shear crack becomes unstable and forms the chip. In our analysis we consider the effect of the vertical crack on the shear crack. The vertical cracks decrease the maximum value of the cutting force T, because the crack interaction increases the SIF, and the unstable growth of the shear crack begins under a lower force T compared to the case without vertical cracks. The surfactant-filled crack, located in front of the shear fracture, transmits normal stress σ_n but not tangential stress τ_{nt}, since the friction between its surfaces decreases significantly due to the effect of the surfactant. The vertical crack is located in the stress field created by the shear crack. The influence of the vertical surfactant-filled crack on the shear fracture is considered a change in SIF. SIF at the tip of the shear fracture is a function of the crack length (a), distance between cracks (l), and applied force T (Fig. 4a). We analyze this problem using the method of path independent integral (Rice, 1968; Cherepanov, 1979). The main equation of this method is given below:

$$\oint_{\Sigma} (Un_1 - \sigma_{ij} n_j u_{i,1}) d\Sigma = 0, \tag{9}$$

where

U is elastic potential per unit volume;
n_j are components of the unit normal to the contour of integration Σ (Fig 4b);
σ_{ij} is the stress field; and
u_i is the displacement and $u_{i,1} = \partial u_i / \partial x_1$.

The contour Σ is divided into parts, and the integral (9) is calculated on each of them.

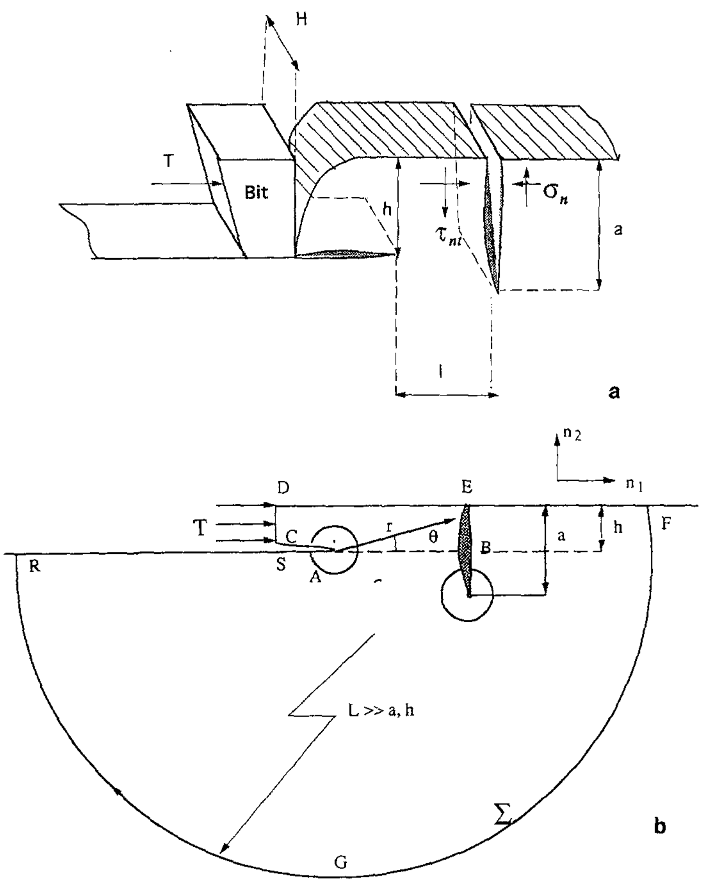

Fig. 4 **a.** Rock cutting with damage zone crack. T is cutting force; σ_n and τ_{nt} are stresses applied to the vertical crack; a is the vertical crack length; H is the bit width; h is the chip thickness; l is the bit width; h is the chip thickness; l is the distance between cracks. **b.** Scheme for calculation of cutting with damage zone crack. T is cutting force; Σ is an integration contour; a is vertical crack length; h is the chip thickness; L is the size of the integration contour; r is the distance from crack tip (A) and points of vertical crack (B) and θ is the corresponding angle; A, B, C, D, E, F, G, R, and S are points on the integration contour; n_1 and n_2 are components of the unit normal to the contour Σ.

Substituting the stresses in Eq. (9), we obtain the expression for the cutting force. The integral equals zero along the half plane boundary (intervals, DE, EF, RS in Fig. 4b) and along the shear fracture because $n_1 = 0$ and $\sigma_{22} = \sigma_{12} = 0$ there. The integral along the curve $\overline{FGR}$ (Fig. 4b) is infinitesimal, because the deformation energy there has an order of $1/L^2$ and the length of this curve is proportional to L. Subsequently, the integral is proportional to $1/L$, and L a large value. Near the ledge (CD in Fig. 4b) the variables are as follows (Cherepanov et al., 1989):

$$\sigma_{11} = -\frac{T}{Hh}; \quad \sigma_{22} = \tau = 0; \quad u_{11} = \frac{\sigma_{11}}{E}; \quad U = \frac{\sigma_{11}^2}{2E}; \quad n_1 = -1; \quad n_2 = 0.$$

The integrals around the crack tips (A and B in Fig. 4b) are equal to $-H\Gamma_0$ and $H\Gamma_1$ respectively. The integrals Γ_0 and Γ_1 correspond to the energy dissipation at the crack tips and equal $\Gamma_i = \frac{K_{II}^2(\text{crack}_i)}{E}$. After elementary transformations, the Eq. (9) can be reduced to the form:

$$\frac{K_{II}^2(A)}{E} - \frac{K_{II}^2(B)}{E} = \frac{T^2}{2EH^2h}. \tag{10}$$

In order to obtain the cutting force as a function of geometrical parameters and fracture toughness, we first, express K_{II} (B) as a function of K_{II} (A). We are interested in the maximum value of the cutting force. At the moment of chip formation the shear crack turns to the free surface and its further propagation is defined by the criterion (1), as has been shown by Hori and Nemat-Nasser (1985) and by Arshon and Staroselsky (1990). To determine SIF K_{II} (B) as a function of $K_{II}(A)$, we employ the following general arguments: force T creates the compressive stress on the crack (B). Because of reduced friction between the sides of the crack, this compressive stress does not generate friction stress and does not influence K_{II} (B), and, therefore, only shear stress acts on the vertical crack boundaries. The shear stress τ on the boundaries of the crack (B) is created by the shear fracture (A), according to the expression

$$\tau = \frac{K_{II}(A)}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right). \tag{11}$$

the distance r and the angle θ are shown in Fig. 4b.
Now we calculate the stress intensity factor $K_{II}(B)$ as a function of the stress intensity factor $K_{II}(A)$ and geometrical parameters by solving the boundary value problem for the vertical crack with boundary conditions (11). As a result of this solution, we obtain the exact form of the interaction effect between horizontal and vertical cracks. This function depends on the ratio between the length of the vertical crack and chip thickness, and its graph is shown in Fig. 5. Substituting the expressions for the stress intensity factors and the fracture criterion for (10), we obtain the maximum value of the force T, which corresponds to the chip formation:

$$T = \frac{1}{\eta} K_{IC}\sqrt{2\,h}\; Hf(Y); \quad Y = \frac{a}{h}, \tag{12}$$

where

η is a coefficient depending on the cutting tool shape;

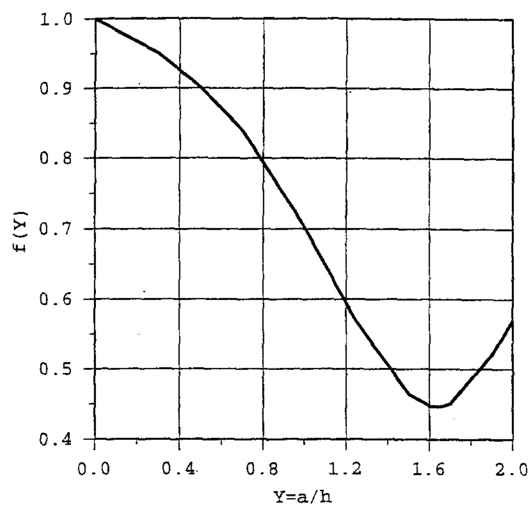

Fig. 5. Function of interaction effect *f(Y)* between the damage zone and shear cracks. The minimum of the function corresponds to the maximum influence of ther vertical crack on the shear crack. $y = a/h$, a is the length of the vertical crack, h is the chip thickness

h is chip thickness;
H is the bit width (Fig. 4a);
a is the length of the vertical crack;
$f(Y)$ is the interaction effect between the damage zone and shear cracks, which is affected by the surfactant.

One can see that the longer the vertical crack, the greater its influence on the chip formation. It should be noted, however, that if the length of crack (B) in Figs. 4 becomes too large, i.e. its tip is too far from the shear crack, the effect of crack interaction decreases. That is why the function $f(Y)$ has a minimum.

The results of the calculations are plotted in Fig. 6. Using the test data (Appendix B), we estimate the changes of relative cutting force (T/T_{dry}) under the influence of the surfactant. We compare the computational results with experimental data (Staroselsky et al. 1991) obtained from the bench test. The cutting force is measured during each bit pass. The surfactant solution is applied only once, when the first layer of rock is removed. Subsequent bit passes are done without surfactant application. The circles in Fig. 6 represent the experimentally determined relative cutting force against the number of removed rock layers. Here T_{dry} is an average cutting force obtained during control cutting of the same block under dry (no surfactant) conditions. To calculate the cutting force, we at first obtain the length of the crack in the damage zone with formulae (1–5). The numerical results appear as crosses in Fig. 6. This analysis supports the hypothesis that the beneficial effects of surfactants are not simply due to the chemical action of the surfactant, but due to a combination of mechanical and chemical action. Also, because of the cracks in the damage zone, the average size of each piece of removed chip should decrease. This is in good agreement with observations on rock cutting under surfactant treatment.

We have to note that all solutions listed above are first order approximations.

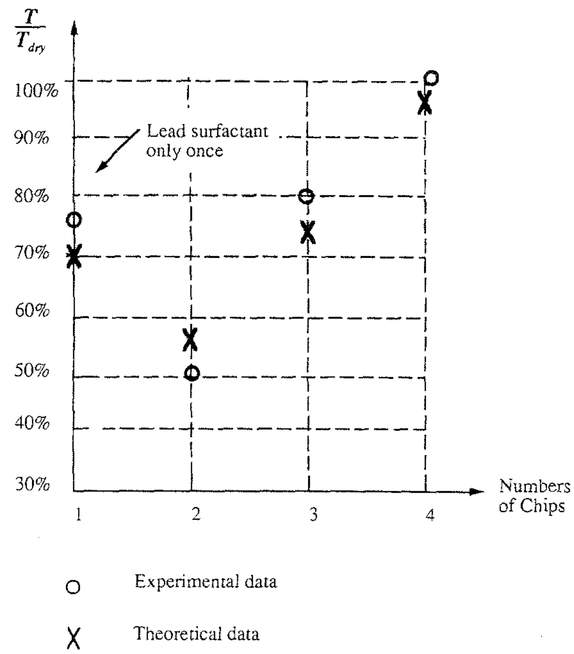

Fig. 6. Relative cutting force T/T_{dry} vs. number of removed chip

Attempts to obtain more exact solutions would lead to tremendous complication of calculations without any significant benefits in explanation of the process.

3. Concluding Remarks

The mechanical model we propose in this paper provides a quantitative explanation of the Rebinder effect during shear/drag bit cutting of rock. This paper analyzes three problems.

First, we examine growth of the hydro-fracture type crack in front of the bit. Second, we estimate the crack propagation behind the bit. These two solutions provide us with information on the development of the damage zone. Third, we determine the influence of the damage zone on the cutting force.

Use of an appropriate surfactant intensifies the development of edge cracks in the damage zone. Such cracks developing in front of the bit tend to grow, facilitating chip formation. Crack lengths induced with the aid of surfactants may reach as much as twice the thickness of the removed chips. Edge cracks in the damage zone do not tend to grow behind the bit, but remain open long enough to allow the surfactant to penetrate into them and fill them until the following cutting passes. As a result, a reduction of the average cutting force up to 50% is obtained. The results of numerical simulations correspond closely to experimental data. One can conclude that the Rebinder effect in rock cutting manifests itself as a combination of two phenomena, namely, a decrease of fracture toughness of the rock and development of the damage zone, which affects subsequent bit passes.

Appendix A

The complete solution for an internal crack normal to the semi-plane boundary under the given load $f(t)$ is given in Savruk's (1981) work:

$$\mathrm{K}_{\mathrm{I}}^{-} - i\,\mathrm{K}_{\mathrm{II}}^{-} = \frac{1}{\sqrt{\pi}}\frac{b^c\sqrt{2c}}{\sqrt{(b^c-d^c)d}}\int_d^b \left[\left(\frac{d}{b}\right)^c f(t)\sqrt{\frac{b^c-t^c}{t^c(t^c-d^c)}} + \frac{c}{2}\left(\frac{E(k)}{K(k)}-\frac{d^c}{t^c}\right)\frac{\sqrt{t^c}F(t)}{t\sqrt{(t^c-d^c)(b^c-t^c)}}\right]dt; \tag{13}$$

$$\mathrm{K}_{\mathrm{I}}^{+} - i\,\mathrm{K}_{\mathrm{II}}^{+} = \frac{1}{\sqrt{\pi}}\frac{b^c\sqrt{2c}}{\sqrt{(b^c-d^c)b}}\int_d^b \left[f(t)\sqrt{\frac{t^c-d^c}{t^c(b^c-t^c)}} - \frac{c}{2}\left(\frac{E(k)}{K(k)}-\frac{d^c}{t^c}\right)\frac{\sqrt{t^c}F(t)}{t\sqrt{(t^c-d^c)\,(b^c-t^c)}}\right]dt;$$

where K^-, K^+ are stress intensity factors in the tips $t = d$ and $t = b$ respectively; $k = \sqrt{1-\left(\frac{d}{b}\right)^c}$; $K(k)$, $E(k)$ are the full elliptic integrals; $c = 2\pi^2/\pi^2 - 4$; $f(t)$ is the load on the crack sides

$$f(t) = \sigma_n - i\tau_{nt}; \quad F(t) = \int_0^t f(\zeta)\quad d\zeta. \tag{14}$$

The Eqs. (13) are so unwieldy that they can not be used for the determination of pressure p_0 and of the final crack length. However, a rough estimate indicates that the size of the closed zone d is small in comparison to the crack length. The asymptotic value of $K_{\mathrm{I}}(d)$ is thus determined by the condition $d/b \rightarrow +0$, and $K_{\mathrm{I}}(d) = 0$ because the stresses at $t = d$ are finite. Solving this equation we obtain the expressions (3) and (5).

Appendix B

The tests to determine the fracture toughness of different sandstones are conducted using the classic four-point bending scheme. Table 2 presents the values of K_{IC} for sandstones treated with aqueous solutions of surfactants.

Table 2.

Rock types	Active medium	K_{Ic} Mpa ,$^{1/2}$	Coefficient of variation	Rock characteristics
Sandstone 1	dry	0.82	10	Co: 66.1 Mpa
	SF-1	0.65	31	To: 6.2 Mpa
	SF-2	0.57	20	Fine grained
	SF-4	0.67	25	
Sandstone 2	dry	1.33	8	Co: 100.6 Mpa
	SF-1	1.00	28	To: 23.9 Mpa
	SF-2	0.92	32	Medium grained
	SF-3	1.12	11	
Sandstone 3	dry	1.05	7	Co: 86.0 Mpa
	SF-2	0.58	12	To: 9.1 Mpa
	SF-3	0.78	24	Fine-medium
	SF-4	1.47	15	Grained
Sandstone 4	dry	0.86	8	Co: 85.3 Mpa
	SF-1	0.74	24	To: 9.1 Mpa
	SF-2	0.72	30	Fine - medium
	SF-3	0.79	13	Grained
	SF-4	0.80	16	
Sandstone 5	dry	1.19	7	Co: 192.7 Mpa
	SF-1	1.11	20	To: 10.2 Mpa
	SF-3	1.15	15	Fine grained
	SF-4	1.02	18	

Table 3.

Surfactant types	pH	Concentration level
SF-1 (sulfonic acid, NaCl, foam generator	8.75	0.3%
SF-2 (sulfonic acid, NaCl)	9.0	0.2%
SF-3 (carboxy methyl cellulose, Na_2CO_3)	10.5	0.15%
SF-4 (katamin, Na_2CO_3)	10.2	0.15%

We have used different surfactants. Table 3 summarizes the pH and concentration levels of them.

Surfactant SF-3 is a high-molecular weight compound and impregnation of the specimen, therefore, occurs slowly. The molecules of anion-active surfactant SF-1 are much smaller and penetrate more easily to the tip of the crack. When the solution was fed directly into the crack, the maximum (20%–25%) reduction of K_{IC} is obtained with an anion-active surfactant with electrolyte additives. A cation-active surfactant (SF-4) is effective for sandstone with closely-packed quartz grains, and an increase of fracture toughness was observed for sandstones with rare (or weak) packing (Sandstone 3 in Table 2).

The increase in the coefficient of variations is apparently associated with the fact that both plastic and brittle properties may appear in rock with different moisture contents that increase or decrease the fracture toughness of the rock respectively.

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