

Sample Exam II M515

- Determine an svd for A (without the use of Matlab) where $A = xy^T$ with $x = [3, 4]^T$ and $y = [-1, 2]^T$.
- Find the matrix P of the orthogonal projection onto the line L in \mathbb{R}^2 that consists of all scalar multiples of the vector $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$. Use P to find the projection of the vector

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ onto the line } L. \text{ Do not use Matlab.}$$

- Fit a trigonometric function of the form $f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t)$ to the data points $(0, -1.5)$, $(\frac{\pi}{2}, -5.5)$, $(\pi, -5.5)$, $(\frac{3\pi}{2}, 0.5)$, using least squares and SVD. Is the solution unique?

- $\mathbb{R}^{2 \times 2}$ with the inner product $\langle A, B \rangle = \text{Tr } A^T B$ is a real inner product space. Let S be the subspace spanned by the matrices $M_1 = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $M_2 = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}$.

Find an orthonormal basis for S . Use the norm $\|A\| = \sqrt{\langle A, A \rangle}$ induced by the inner product.

- For the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 2 & 4 & 6 \end{bmatrix}$

a) Use Matlab to find an SVD for A . Identify U , Σ , V , U_1 , U_2 , V_1 , V_2 , and S (notation from Laub).

b) Name and find bases for the four fundamental subspaces of A . Are they orthonormal bases?

c) Given a vector $v \in \mathbb{R}^4$, which fundamental subspaces could v be in? Must it be in one of them? Could it be in more than one of them?

d) What is the rank of A ?

e) What is the nullity (dimension of the null space) of A ?

f) Is A onto? Is it 1-1? Explain.

- For the matrix $A = \begin{bmatrix} 1 & 2.01 & 3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$

a) Find the 1, 2 and ∞ norms of A without using the Matlab norm command.

b) Find the condition number of A with respect to the 1, 2, and ∞ norms.

c) We want to solve the equation $Ax = b$ for x , where A and b are matrices whose values have been obtained by measurement. If there is a 1% error in b , how big could the error in x possibly be?