M515 Sample Exam I

1. Use row operations to reduce $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$ to upper triangular form, and then determine $\det(A)$ based on the result. Do not use Matlab for this one.

2. Let $v_1 = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$. Find a third vector v_3 for so that $\{v_1, v_2, v_3\}$ forms an orthogonal set. Use that orthogonal set to find an orthonormal set $\{w_1, w_2, w_3\}$. Finally form an orthogonal matrix Q from the orthonormal set by creating a matix with the orthonormal vectors as columns, and show that it is in fact orthogonal.

3. Find a basis for each subspace of R^4:

a) All vectors whose components are equal.

b) All vectors whose components add to zero.

4. Which of the following subsets of $R^{2\times 2}$ are subspaces of $R^{2\times 2}$? Either find a counter example or prove that it is a subspace.

a) The 2x2 matrices whose entries are all greater than or equal to 0

b) The 2x2 matrices with all zeros in the second row

c) The non-invertible (singular) 2x2 matrices

5. Polynomials of degree less than or equal to two of the form $p_0 + p_1 x + p_2 x^2$ form a vector space of dimension 3, and the polynomials 1, x, $2x^2 - 1$ form a basis for this vector space. Find the components of the polynomial $2 + 3x + 4x^2$ with respect to this basis.

6. Let $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Find bases for the null space of A and for the range of A. Are these spaces equal? Is this true in general? If so, prove it, and if not, provide a counterexample.

7. a) Find a basis for the range of $M = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 1 & 5 & 1 \end{bmatrix}$

b) If possible, find k so that the vector $[1, 1, k]^T$ is in the range of M.

c) Find a basis for the null space of M.

d) If possible, find k so that the vector $[1, 1, k]^T$ is in the null space of M.

e) What are the dimensions of the range and null space?

8. Find bases for $R(M)^\perp$ and $N(M)^\perp$ for the matrix in 7.