Homework Set 2

Due Thurs Nov 5

1. Find a singular value decomposition of $A = \begin{bmatrix} 2 & 3 & -6 & 3 & 2 \\ 10 & 9 & -6 & 8 & 5 \\ 7 & 2 & 1 & 9 & 8 \\ 8 & 6 & 0 & 5 & 3 \end{bmatrix}$ in two ways.

First use the definition of SVD given in class (that is, using eigenvalues and eigenvectors of A^TA, using the Matlab eig command), and again using the Matlab svd command. Show that each SVD is correct (that is, show that $A = U\Sigma V^T$, that U and V are orthogonal, and that Σ is diagonal). Finally for each SVD find the matrices U_1, S, V_1 and show that $A = U_1 S_1 V_1^T$.

2. Find a digital image (of your own, or from the web). Choose an image whose size is at least 600 pixels on each side. Use the SVD to compress the image, by eliminating all but the largest n% of the singular values, and compare to the original image. Find the smallest value of n for which the image appears about the same as the original image (at arms length). What is your compression ratio? Include in your report the original image. Also include the compressed image at the value of n% that you decide on, as well as values both greater and less than that value (so that the reader can see why you choose that value).

3. a) Find orthonormal bases for the four fundamental subspaces of the linear transformation $A = \begin{bmatrix} 2 & -1 & 1 & 1 \\ -3 & 0 & -6 & 12 \\ 7 & -2 & 8 & -10 \\ 4 & 3 & 17 & -43 \end{bmatrix}$ using rref and gram schmidt.

b) Repeat part a) using the svd command. Did you get the same bases? Is it a problem if they are not the same? Explain.

c) Consider the vector $v = [-1\ -1\ -5\ 13]^T$. This vector lies in one of the fundamental subspaces of A. Determine which one, and find the components of this vector with respect to the basis you got from part b).

4. Extra credit: Do Laub, Exercise 1, Chapter 3 (p 27).