M515 Homework Set 1

Due Tues, Sept 22

1. Use Matlab to do the following. Create a 8×8 random (uniform on $[-4,5]$) integer matrix A and a 8×1 random (uniform on $[-4,5]$) integer matrix b (i.e. a column vector).

a) Find Ab directly (Matlab matrix multiply command)

b) Find Ab by explicitly dotting each row of A with b, and show it is the same result as in a).

c) Find Ab by taking an appropriate linear combination of the columns of A, and show it is the same result as in a) and b).

2. Use Matlab to show that $AB = [Ab_1, Ab_2, \ldots, Ab_n]$ where $B = [b_1, b_2, \ldots, b_n]$. Use random matrices of size 4×5 and 5×6 for A and B respecively.

3. Exercise 3 on page 6 of Laub.

Hint: Partition the matrix $I - xy^T$ into a block matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$, with A being just a single element (a 1×1 matrix). Then use row operations to turn D into the identity (add appropriate multiples of row 1 to the other rows). Finish by using determinant property 17 on page 5 of Laub.

Note: You may want to try the 3×3 case before you try the general case. Partial credit if you can only do the 3×3 case.

4. Do exercise 5b on page 6 of Laub. You may assume that 5a on page 6 of Laub is true without having to prove it.

Hint: Let $U = A$ and $V = B^T$ so that $B = V^T$ and hence we are now trying to prove that $Tr(UV^T) = Tr(V^TU)$. Next show for column vectors u, v of the same length that $u^Tv = Tr(uv^T)$ (i.e. inner product = Trace(outer product)). Then use Theorem 1.3 on $Tr(UV^T)$, and use the dot product definition of matrix multiplication on $Tr(V^TU)$ to show that they are equal. Remember that dot product of vectors is commutative.

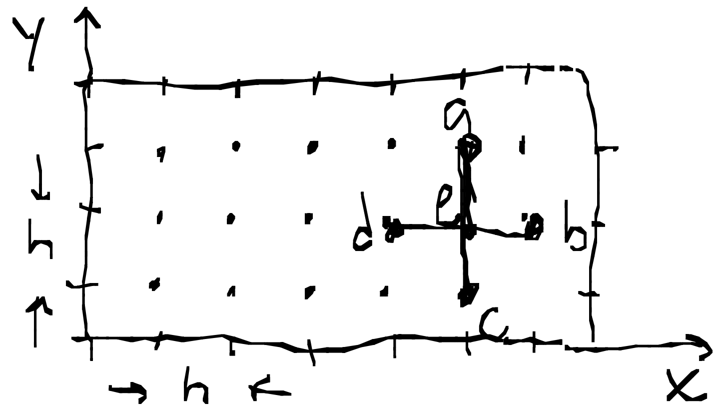
5. Put each system of equations into matrix form $Ax = b$, and solve each system using rref in Matlab. If there are no solutions say so, and if there are infinitely many, give a parametric representation. Also, determine the rank of matrix A for each. How is the rank related to the number of solutions?

a) $$\begin{bmatrix} x - 2y + 3z - w = 2 \\ 3x + y + z - w = 5 \\ 4x + 5y - z = 0 \\ -2x - y - z + 5w = 3 \end{bmatrix}$$

b) $$\begin{bmatrix} x - 2y + 3z - w = 2 \\ 3x + y + z - w = 5 \\ 4x + 5y - z = 0 \\ 7x + 11y - 2z + w = 3 \end{bmatrix}$$

c) $$\begin{bmatrix} x - 2y + 3z - w = 2 \\ 3x + y + z - w = 5 \\ 4x + 5y - z = 0 \\ 7x + 11y - 2z + w = -8 \end{bmatrix}$$

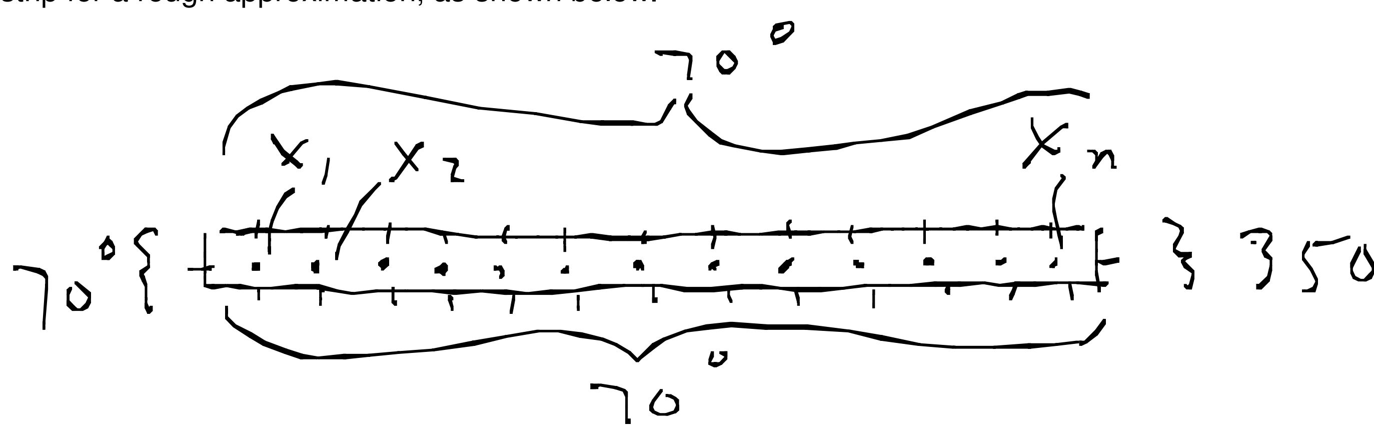
6. If the temperature on the boundary (edges) of a thin flat plate are specified, then Laplace's partial differential equation (in two dimensions) can be used to determine the temperature at any interior point of the plate. When Laplace's equation is solved using finite differences, one lays out grid points (nodes) on the surface of the plate and solves for the approximate temperature at each node. The simplest case is when the plate is rectangular; then one can choose a rectangular array of nodes to create a rectangular mesh.



If the spacing h is the same in the x and y directions, then Laplace's equation implies that the temperature at each interior node is the average of the temperatures at each of the four neighboring nodes. Thus $e = \frac{1}{4}(a+b+c+d)$ or $4e = a+b+c+d$ from the sketch above (letters represent temperatures at that node).

We will apply this idea to a long thin rectangular strip of metal. Say that the strip is part of a circuit board, and that all of the edges of the strip are normally kept at 70^o (and so all the interior points would also be 70^o). Now if one of the short edges is in contact with a circuit element that overheats, say to 350^o, how are the interior points of the strip affected?

Since the strip is long and thin, we will put single nodes down the center of the strip for a rough approximation, as shown below.



The temperatures at the nodes down the middle are $x_1, x_2, \ldots, x_n$. For $n = 4$ nodes in the interior, the equations would be $4x_1 = x_2 + 210$, $4x_2 = x_1 + x_3 + 140$, $4x_3 = x_2 + x_4 + 140$, $4x_4 = x_3 + 490$ (do you see why?).

Set up the equations for this problem using $n = 50$ nodes down the middle. Put the equations into $Ax = b$ matrix form, and solve using Matlab. To create the

50×50 matrix A, you may want to make use of the ones(m,n) command, the triu and/or tril commands, the eye(n) command, or any other Matlab command you feel is helpful. How far down the strip does the overheated element affect the interior temperature of the strip (to Matlab's default precision)?