Note Title 9/27/2009

1)

```
> A=floor(10*(rand(8)-0.5))+1
A =

 4  2 -3 -3  1  1  4 -2
 1  5  5 -4  3  1  5  1
-2 -3 -4 -1  5 -2 -4  3
-4 -1 -3  5  0 -1  1  1
 5 -2  3  4 -4  5  3  1
 4  5  3 -4 -3 -4  5  4
 3  3 -2  4  1 -2  3 -2
 3 -1  5 -3 -3 -4  2 -4


> b=floor(10*(rand(8,1)-0.5))+1
b =

-2
 5
 1
 5
-2
 1
 4
 0
```

a)

```
> A*b
ans =

 -1
 23
-48
 28
 28
 22
 35
-11
```

b)

```
> [A(1,:)*b;A(2,:)*b;A(3,:)*b;A(4,:)*b;A(5,:)*b;A(6,:)*b;A(7,:)*b;A(8,:)*b]
ans =

 -1
 23
-48
 28
 28
 22
 35
-11
```

c)

```
> b(1)*A(:,1)+b(2)*A(:,2)+b(3)*A(:,3)+b(4)*A(:,4)+b(5)*A(:,5)+b(6)*A(:,6)+b(7)*A(:,7)+b(8)*A(:,8)
ans =

 -1
 23
-48
 28
 28
 22
 35
-11
```

Note Title 9/27/2009

2)

```
> A=rand(4,5)
A =

   0.356273   0.942772   0.319577   0.603306   0.435611
   0.465872   0.803947   0.271860   0.983701   0.512123
   0.311310   0.995021   0.245254   0.949265   0.803568
   0.061958   0.177472   0.685469   0.374224   0.970725


> B=rand(5,6)
B =

   0.784301   0.326912   0.640544   0.984339   0.348838   0.192290
   0.063107   0.924149   0.629024   0.438841   0.925304   0.537977
   0.514666   0.033309   0.684992   0.252643   0.894820   0.385333
   0.186759   0.170506   0.240126   0.352948   0.845137   0.981421
   0.215273   0.725660   0.387589   0.244500   0.414393   0.654547


> A*B
ans =

   0.70984   1.41735   1.35385   1.16460   1.97299   1.57607
   0.85000   1.44368   1.42504   1.35247   2.19326   1.92748
   0.78345   1.77446   1.53269   1.33657   2.38400   2.14727
   0.69144   0.97532   1.08696   0.68147   1.51773   1.37418


> [A*B(:,1),A*B(:,2),A*B(:,3),A*B(:,4),A*B(:,5),A*B(:,6)]
ans =

   0.70984   1.41735   1.35385   1.16460   1.97299   1.57607
   0.85000   1.44368   1.42504   1.35247   2.19326   1.92748
   0.78345   1.77446   1.53269   1.33657   2.38400   2.14727
   0.69144   0.97532   1.08696   0.68147   1.51773   1.37418
```

3) $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $\quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

$$I - xy^T = \begin{bmatrix} 1-x_1y_1 & -x_1y_2 & -x_1y_3 & \cdots & -x_1y_n \\ -x_2y_1 & 1-x_2y_2 & -x_2y_3 & \cdots & -x_2y_n \\ \vdots & & & & \vdots \\ -x_ny_1 & -x_ny_2 & \cdots & -x_ny_{n-1} & 1-x_ny_n \end{bmatrix}$$

$-\frac{x_2}{x_1} \cdot R1 + R2$, $-\frac{x_3}{x_1} \cdot R1 + R3$, $\ldots$, $-\frac{x_n}{x_1} \cdot R1 + R_n$

(NONE OF THESE CHANGE DET)

$$\rightarrow \begin{bmatrix} 1-x_1y_1 & -x_1y_2 & -x_1y_3 & \cdots & -x_1y_n \\ -\frac{x_2}{x_1} & 1 & 0 & \cdots & 0 \\ -\frac{x_3}{x_1} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -\frac{x_n}{x_1} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

NOW USE PROPERTY 17 ON P. 5 OF LAUB

SO $\det(I - xy^T) = \det(D) \cdot \det(A - BD^{-1}C)$

($D = I$ SO $D^{-1} = I$ AND $\det(D) = 1$)

$= \det(A - BC)$

$$= 1 - x_1 y_1 - (x_2 y_2 + x_3 y_3 + \cdots + x_n y_n)$$
$$= I - x^T y$$

4) LET $u = \begin{bmatrix} x_i \\ \vdots \\ x_n \end{bmatrix}$ $\quad v = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ $\quad u, v \in \mathbb{R}^n$

THEN $TR(uv^T) = TR \begin{bmatrix} x_1 y_1 & x_2 y_1 & \cdots & x_n y_1 \\ x_1 y_2 & x_2 y_2 & \cdots & x_n y_2 \\ \vdots & & & \vdots \\ x_1 y_n & x_2 y_n & \cdots & x_n y_n \end{bmatrix}$

$$= x_1 y_1 + x_2 y_2 + \cdots + x_n y_n = u^T v$$

NOW LET $U = A$ AND $V = B^T$ SO THAT $B = V^T$

THEN $U = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_n \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times n}$

$V = \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times n}$

FROM THM 1.3

$$TR(UV^T) = TR\left(\sum_1^n u_i v_i^T\right)$$
$$= \sum_1^n TR(u_i v_i^T) \quad \text{(FROM PROB 5a)}$$
$$= \sum_1^n u_i^T v_i \quad \text{FROM ABOVE}$$

ALSO $TR(V^T U) = TR \begin{bmatrix} v_1^T u_1 & v_1^T u_2 & \cdots & v_1^T u_n \\ v_2^T u_1 & v_2^T u_2 & \cdots & v_2^T u_n \\ \vdots & & & \\ v_n^T u_1 & v_n^T u_2 & \cdots & v_n^T u_n \end{bmatrix}$

(USING THE USUAL "DOT PRODUCT" DEFINITION OF MATRIX MULTIPLICATION)

$$= \sum_1^n v_i^T u_i = \sum_1^n u_i^T v_i$$

5) a)

```
> A=[1 -2 3 -1;3 1 1 -1;4 5 -1 0;-2 -1 -1 5]
A =

  1  -2   3  -1
  3   1   1  -1
  4   5  -1   0
 -2  -1  -1   5


> b=[2;5;0;3]
b =

  2
  5
  0
  3


> rref([A b])
ans =

  1.00000   0.00000   0.00000   0.00000   4.54237
  0.00000   1.00000   0.00000   0.00000  -4.32203
  0.00000   0.00000   1.00000   0.00000  -3.44068
  0.00000   0.00000   0.00000   1.00000   0.86441
```

SO ONE SOLUTION $x = 4.54237$ $y = -4.32203$

RANK(A) = 4 $z = -3.44068$ $w = 0.86441$

b)

```
> A(4,:)=[7 11 -2 1]
A =

  1  -2   3  -1
  3   1   1  -1
  4   5  -1   0
  7  11  -2   1


> rref([A b])
ans =

 1.00000   0.00000   0.00000  -0.53846   0.00000
 0.00000   1.00000   0.00000   0.46154   0.00000
 0.00000   0.00000   1.00000   0.15385   0.00000
 0.00000   0.00000   0.00000   0.00000   1.00000
```

LAST EQUATION IS $0 = 1$ SO NO SOLUTIONS

RANK(A) = 3

c)

```
> b(4)=-8
b =

   2
   5
   0
  -8

octave-3.2.0.exe:20:C:\Octave\3.2.0_gcc-4.3.0\bin
> rref([A b])
ans =

   1.00000   0.00000   0.00000  -0.53846   4.07692
   0.00000   1.00000   0.00000   0.46154  -3.92308
   0.00000   0.00000   1.00000   0.15385  -3.30769
   0.00000   0.00000   0.00000   0.00000   0.00000
```

RANK(A) = 3

ZERO ROW $\rightarrow$ ∞ MANY SOLUTIONS

$$z = -3.30769 - 0.15385w$$

$$y = -3.92308 - 0.46154w$$

$$x = 4.07692 + 0.53846w$$

IF RANK(A) = 4 = #VARIABLES $\rightarrow$ 1 SOLUTION

IF RANK(A) < 4 = #VARIABLES $\rightarrow$ 0 OR ∞ SOLUTIONS

5) THE EQUATIONS ARE

$$4x_1 = 70+70+70+x_2$$
$$4x_2 = 70+70+x_1+x_3$$
$$4x_3 = 70+70+x_2+x_4$$
$$\vdots$$
$$4x_{49} = 70+70+x_{48}+x_{50}$$
$$4x_{50} = 70+70+350+x_{49}$$

50

$$A = \begin{bmatrix} 4 & -1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 4 & -1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 4 & -1 & 0 & \cdots & 0 \\ & & & \vdots & & & \\ 0 & \cdots & & & -1 & 4 & -1 \\ 0 & \cdots & & & 0 & -1 & 4 \end{bmatrix} \qquad A \in \mathbb{R}^{50\times 50}$$

$$b = \begin{bmatrix} 210 \\ 140 \\ 140 \\ \vdots \\ 140 \\ 490 \end{bmatrix} \qquad b \in \mathbb{R}^{50}$$

```
> A=-1*ones(50,50);

> A=triu(A,-1);

> A=tril(A,1);

> A=A+5*eye(50,50);

> A(1:10,1:10)
ans =

   4  -1   0   0   0   0   0   0   0   0
  -1   4  -1   0   0   0   0   0   0   0
   0  -1   4  -1   0   0   0   0   0   0
   0   0  -1   4  -1   0   0   0   0   0
   0   0   0  -1   4  -1   0   0   0   0
   0   0   0   0  -1   4  -1   0   0   0
   0   0   0   0   0  -1   4  -1   0   0
   0   0   0   0   0   0  -1   4  -1   0
   0   0   0   0   0   0   0  -1   4  -1
   0   0   0   0   0   0   0   0  -1   4

>
```

```
> b=140*ones(50,1);

> b(1,1)=210;

> b(50,1)=490;

> b'
ans =

 Columns 1 through 19:

   210   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140
140

 Columns 20 through 38:

   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140   140
140

 Columns 39 through 50:

   140   140   140   140   140   140   140   140   140   140   140   490
```

```
> sol=rref([A b]);

> sol(50,:)
ans =

 Columns 1 through 9:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 10 through 18:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 19 through 27:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 28 through 36:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 37 through 45:

    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000    0.00000

 Columns 46 through 51:

    0.00000    0.00000    0.00000    0.00000    1.00000  145.02577


> sol(:,51)'
ans =

 Columns 1 through 11:

   70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000
70.000

 Columns 12 through 22:

   70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000
70.000

 Columns 23 through 33:

   70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.000
70.000

 Columns 34 through 44:

   70.000    70.000    70.000    70.000    70.000    70.000    70.000    70.001    70.002    70.007
70.028

 Columns 45 through 50:

   70.104    70.387    71.443    75.387    90.103   145.026
```

OVERHEATED ELEMENT AFFECTS
10 NODES (TO 3 DECIMAL PLACES)