Note Title 10/8/2009

1. $\det(BA) = \det(B)\det(A) = 14$

$\det(-3A) = (-3)^4 \det(A) = 162$

$\det(A^T) = \det(A) = 2$

$\det(B^{-1}) = 1/\det(B) = 1/7$

$\det(B^9) = (\det(B))^9 = 7^9$

2. $\text{RREF}(A) = \begin{bmatrix} 1 & 0 & 0 & 5 & 3.5 \\ 0 & 1 & 0 & -6 & -2.5 \\ 0 & 0 & 1 & -2.5 & -1.5 \end{bmatrix}$

a) USE $v, w, x, y, z$

$$v = -5y - 3.5z$$
$$w = 6y + 2.5z$$
$$x = 2.5y + 1.5z$$

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{bmatrix} -5y - 3.5z \\ 6y + 2.5z \\ 2.5y + 1.5z \\ y \\ z \end{bmatrix} = y\begin{bmatrix} -5 \\ 6 \\ 2.5 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -3.5 \\ 2.5 \\ 1.5 \\ 0 \\ 1 \end{bmatrix}$$

BASIS FOR $N(A)$

b) $1^{st}$ 3 COLS OF A

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix} \right\} = \text{BASIS FOR } R(A)$$

c) RANK = 3 (# ROWS AFTER RREF)

d) $\dim(R(A)) = 3$ $\dim(N(A)) = 2$

e) IS ONTO, NOT 1-1

3. $\text{RREF} \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ SO LIN INDEP

ANY 2 LIN INDEP VECTORS SPAN $\mathbb{R}^2$
SO IS BASIS

$$\begin{pmatrix} 5 \\ 1 \end{pmatrix} = x \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\text{RREF} \begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \qquad \begin{matrix} x = -1 \\ y = 2 \end{matrix}$$

$$\text{SO} \begin{pmatrix} 5 \\ 1 \end{pmatrix} = -v_1 + 2v_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}_v$$

4. a) NOT SUBSPACE

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad v_1 + v_2 = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$$

NOT ALL DIFFERENT

b) NOT SUBSPACE

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad v_1 + v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

NOT UNIT

c) IS SUBSPACE

IF $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = v_1$ IS IN $V$ AND $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = v_2$ IS IN $V$

THEN $2x_1 - 3y_1 + z_1 = 0$ AND $2x_2 - 3y_2 + z_2 = 0$

ADD EQUATIONS: $2x_1 + 2x_2 - 3y_1 - 3y_2 + z_1 + z_2 = 0$

$$2(x_1 + x_2) - 3(y_1 + y_2) + (z_1 + z_2) = 0$$

SO $v_1 + v_2$ IS IN $V$

ALSO, MULTIPLY 1ST EQ BY $\alpha$:

$$\alpha(2x_1 - 3y_1 + z_1) = 0$$

SO $2(\alpha x_1) - 3(\alpha y_1) + (\alpha z_1) = 0$

SO $\alpha v_1$ IS IN $V$

5) a) ANY SKEW SYMMETRIC CAN BE WRITTEN AS $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$$= a\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

BASIS

dim = 3

b) ANY UPPER TRIANGULAR IS

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} = a\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

BASIS

$$+ d\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + e\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + f\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

dim = 6

6) LET $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -5 & -1 \\ -8 & 7 \end{bmatrix}$ THEN $R(A) = \text{SPAN}\{v_1, v_2\}$

$$R(A)^{\perp} = N(A^T)$$

$$A^T = \begin{array}{cccc} x & y & z & w \\ \end{array}$$
$$A^T = \begin{bmatrix} 1 & 0 & -5 & -8 \\ 0 & 1 & -1 & 7 \end{bmatrix}$$ ALREADY IN RREF

$x = 5z + 8w$

$y = z - 7w$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 5z + 8w \\ z - 7w \\ z \\ w \end{bmatrix}$$

$$= z\begin{pmatrix}5\\1\\1\\0\end{pmatrix} + w\begin{pmatrix}8\\-7\\0\\1\end{pmatrix}$$

BASIS