Practice Exam II M344

SHOW ALL WORK!

1. Find 4 terms of a Fourier cosine series for $f(x) = x^2 - 3\cos(2x)$ on the interval $0 \le x \le \pi$. Sketch the graphs of both $f(x)$ and the cosine series together on the interval $-2\pi \le x \le 2\pi$.

2. A rectangular flat plate is 5 inches long on the bottom and 3 inches on the side. The left, right and bottom edges are kept at 0 degrees. The temperature of the top edge is given by $f(x) = \sin(2\pi x)$ where x is the distance from the left edge. After a long period of time, the temperature at the point that is 1 inch from the left edge and 1 inch above the bottom edge is measured. What is it?

3. Find the eigenvalues (possible K values) and eigenfunctions (possible $X(x)$ functions) for the boundary value problem

$$X''(x) + KX(x) = 0, X(0) = 0, X'(1) = 0.$$

4. A one foot rod is heated in such a way that the temperature at a point x feet away from the left end is given by $\cos(\pi x) + 2\cos(9\pi x)$. The ends are kept insulated. Find the temperature of the rod at a point 0.25 feet from the left end point after 5 seconds. Assume $\beta = 1$.

5. Solve the following boundary/initial value problem (Fourier series method). Estimate $u(0.25, 0.5)$ using three terms of the series (n=1, 2, 3 whether the terms are zero or not). What does this number represent in the context of the "typical" application of this equation?

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, u(0,t) = 0, u(1,t) = 0, u(x,0) = \sin(\pi x), \frac{\partial u}{\partial t}(x,0) = x(1-x)$$