Practice Exam I M344 Advanced Engineering Mathematics

SHOW ALL WORK!

Classify each differential equation below as to whether it is linear or nonlinear, and if linear, find all singular points and classify the real-valued ones as regular or irregular. If nonlinear, use the method of section 8.1 to find three terms of a Taylor polynomial solution in terms of the initial conditions $y(0)$ and $y'(0)$.

Next, if a closed form solution exists (non-series solution, including special functions), give that solution.

Next, if a closed form solution does not exist, determine if a power series solution about the point $x_0 = 0$ would make sense (including Frobenius method if needed). If not, explain why not.

Next, if a power series solution at $x_0 = 0$ does make sense, find three non-zero terms (use Frobenius if necessary). Find the (minimum) radius of convergence for each power series solution.

Finally, determine whether or not you need a second linearly independent solution. If so, give the form of the second solution.

1. $9x^2y'' + 9xy' + (9x^2 - 16)y = 0$
2. $(x^2 + 1)y'' + y = 0$
3. $y'' + yy' = 0$
4. $x^2y'' + 3xy' + y = 0$
5. $xy'' + y' - 4y = 0$
6. $x^3y'' + 3xy' + y = 0$