Note Title 10/6/2009

a) $y(0) = a \quad y'(0) = b \quad y''(0) = -4\sin(y(0))$
$= -4\sin(a)$

$y''' = -4\cos(y)\, y'$

$y'''(0) = -4\cos(a)\cdot b$

$$y = a + bx - \frac{4\sin(a)}{2}x^2 - \frac{4\cos(a)b}{6}x^3 + \cdots$$

b) BESSEL $\quad \nu^2 = 4 \quad \nu = \pm 2$

$y = c_1 J_2(x) + c_2 Y_2(x)$

c) CAUCHY-EULER $\quad r(r-1) + 3r + 2 = 0$
$r^2 + 2r + 2 = 0$

$r = -1 \pm i$

$$y = c_1 x^{-1}\cos(\ln x) + c_2 x^{-1}\sin(\ln x)$$

2) a) $y'' + \dfrac{5}{x(x-1)^2}y' + \dfrac{3}{x^2(x-1)^2}y = 0$

$x = 0,\ x = 1$ SING PTS

FOR $x = 0$; $\quad p = \dfrac{5}{x(x-1)^2} \quad p_0 = xp = \dfrac{5}{(x-1)^2}$

$\to 5$ AS $x \to 0$

$$q = \frac{3}{x^2(x-1)^2} \qquad q_0 = x^2 q = \frac{3}{(x-1)^2} \to 3 \text{ AS } x \to 0$$

REGULAR SING PT

FOR $x = 1$: $p = \dfrac{5}{x(x-1)^2}$ $\quad p_0 = \dfrac{5}{x(x-1)^2} \cdot (x-1)$

$$= \frac{5}{x(x-1)} \to \infty$$

UNDEFINED AS $x \to 1$

IRREGULAR SING PT

POWER SERIES IS POSSIBLE AT $x = 0$

INDICIAL EQ $\quad r(r-1) + 5r + 3 = 0$

$$r^2 + 4r + 3 = 0$$

$$r = \frac{-4 \pm \sqrt{16-12}}{2} = -2 \pm 1 \qquad r_1 = -1, \quad r_2 = -3$$

$$y_1 = x^{-1} \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n x^{n-1}$$

SECOND SOLUTION : $r_1 - r_2 = 2$ IS

POSITIVE INTEGER SO

$$y_2 = C_1 y_1 \ln x + \sum_{n=0}^{\infty} b_n x^{n-3}$$

RADIUS OF CONV IS $R = 1$

(DISTANCE TO NEXT SING POINT

WHICH IS AT $x = 1$)

5) $y'' + \frac{3x}{x-1} y' + \frac{1}{(x-1)^2} y = 0$

$x=1$ IS ONLY SING PT

$p = \frac{3x}{x-1}$ $\quad p_0 = \frac{3x}{x-1}(x-1) = 3x \to 3$ AS $x \to 1$

$q = \frac{1}{(x-1)^2}$ $\quad q_0 = \frac{1}{(x-1)^2}(x-1)^2 = 1 \to 1$ AS $x \to 1$

REGULAR SING PT

$x=0$ IS ORDINARY, SO SERIES SOL OK

$$y = \sum_{n=0}^{\infty} a_n x^n$$

RADIUS OF CONV IS $R=1$

(DISTANCE TO NEAREST SING PT)

3) $y = \sum_0 a_n x^n$ $\qquad x^2 y = \sum_0 a_n x^{n+2}$

$y' = \sum_1 a_n n x^{n-1}$

$$\sum_{n=1} a_n n x^{n-1} + \sum_{n=0} a_n x^{n+2} = 0$$

$k = n-1$ $\qquad k = n+2$

$n = k+1$ $\qquad n = k-2$

$$\sum_{k=0} a_{k+1}(k+1)x^k + \sum_{k=2} a_{k-2}\,x^k = 0$$

$k=0 \qquad a_1(1)=0 \qquad a_1=0$

$k=1 \qquad a_2(2)=0 \qquad a_2=0$

$k=2,3,\ldots \qquad a_{k+1}(k+1)+a_{k-2}=0$

$$a_{k+1} = -\frac{a_{k-2}}{k+1}$$

$y(0)=2$ so $a_0=2 \qquad y=2+0x+0x^2$

$k=2: \qquad a_3 = -\frac{a_0}{3} = -\frac{2}{3}$

$k=3: \qquad a_4 = -\frac{a_1}{4} = 0 \qquad k=4: \quad a_5 = -\frac{a_2}{5} = 0$

$k=6: \quad a_6 = -\frac{a_3}{6} = -\frac{1}{6}\left(-\frac{2}{3}\right) = \frac{1}{9}$

$$y = 2 - \frac{2}{3}x^3 + \frac{1}{9}x^6 + \cdots$$