The use of the visual-spatial intelligence in the solution of elementary physics problems.

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Abstract

The process of transforming a word problem, the only kind in physics, to a mathematical representation of that problem uses several of Howard Gardner's Multiple Intelligences. Traditionally, only the verbal-linguistic and logical-mathematical intelligences have been emphasized. However, the authors have found that solving physics problems requires the ability to visualize the problem statement into a mental video. Not only does this process entail the visual-spatial intelligence but also the kinesthetic as the student needs to have a sense of how the physical world works. This paper addresses visual techniques that aid in the solution of mechanics problems in freshman physics courses. First is the use of multiple sketches to enhance visualization of the problem. Most texts suggest "sketch" the problem with the implication of a single sketch. When the problem is time-dependent, it is appropriate most of the time to provide two or more sketches. These sketches are snapshots in time taken from the mental video of the problem at hand. Another is the use of tables for data organization and recognition. Creating a visual organization of information makes it much easier to see what to do next for the visual-spatial student. Both are visual tools to solve analytical problems. Several examples of application of these methods are presented.

Introduction

The students at Ward College of Technology at the University of Hartford can major in Architectural, Audio, Electronic, Computer, and Mechanical Engineering Technology. They are thus a diverse group, with different abilities. We have found that our Architectural Engineering Technology (AET) students, for example, tend to process information visually, whereas the Audio Engineering Technology (AUET) students process information aurally and the Mechanical Engineering Technology (MET) students process information kinesthetically. To develop strategies for teaching mathematics and writing to this diverse group, strategies that don’t rely on the traditionally emphasized verbal-linguistic and logical-mathematical intelligences, but that strengthen them, we turned to Howard Gardner’s theory of multiple intelligences. The theory helped to validate our initial instinct that our students handle information differently and provided us with resources for our teaching. We have been using strategies informed by Gardner’s theory for two years now. We have applied them to our freshman technical writing course, taught by Ms. Segal, and to our freshman pre-calculus class, taught by Dr. Townsend.

Dr. Townsend (hereafter "I") also teaches a section of algebra-based physics for the College of Arts & Sciences. That section may include Ward College students in addition to biology, premed students, and criminal justice majors. The students are typically not interested specifically in the terse language of mathematics, which is, after all, the grammar of physics. But they are virtually all required to take the course. The issue, once again, is diversity, teaching the solution of word problems in physics to students who are not necessarily strong in verbal-linguistic and logical mathematical intelligences, but who must learn the material and strengthen those two intelligences at the same time. Taking a fresh approach to the teaching of this course, based upon the authors' experiences at Ward, seemed not only appropriate, but mandatory. In this paper we present the results of this effort.

Because many traditional texts, including Serway[1], tell students to solve physics problems by drawing a diagram of the problem, we agreed that strategies involving the visual-spatial intelligence might work well. However, I noted that the suggested single diagram is often insufficient, particularly in problems involving the evolution of the system over time. Mental videos and multiple diagrams may be necessary to fully describe a problem and aid the student in its solution. Further, I realized that a strategy I often use to solve problems I am working on, the data table, is a visual-spatial tool that can help students work out a problem.

What follows is a demonstration of these methods of visually organizing information to solve physics problems.

Kinematics

Introductory kinematics can be very confusing to the new physics students. It is the first topic covered in the course. There are new concepts and new symbols. It is confusing to know which formula to use and what data the problem is actually specifying. In order for the students to get started, I start the course with several problems concerning average velocity, which involves only one formula, $\Delta x = v_{ave}t$. Even so, the students have trouble figuring out how to put the data into the formula.

The next step is to solve problems with constant acceleration. These problems include several types of data and several choices of the appropriate equation(s) to use. Most students have equations and calculations splattered everywhere. To help them reduce the visual (and mental) clutter, I teach them to use tables of both equations and data. The equation table below provides a visually tidy, easy reference for the student.

Kinematic equations in 1D

Equation	Information Given by Equation	Equation parameters				
		Δx	v_0	v	a	t
(1) $\Delta x = v_0 t + \frac{1}{2} a t^2$	Displacement as a function of time	√	√		√	√
(2) $v = v_0 + at$	Velocity as a function of time		√	√	√	√
(3) $\Delta x = \frac{1}{2}(v_0 + v)t$	Displacement as a function of velocity	√	√	√		√
(4) $v^2 = {v_0}^2 + 2a\Delta x$	Velocity as a function of displacement	√	√	√	√	

Each day I put column one on the board as a reference so as the students help me solve an in-class problem, they can glance over the list of equations to tell me which one to use. Note that each equation contains four of the five kinematics parameters. so if three values are known, the fourth can be found. Just pointing out this simple fact has helped many students focus on which equation to use. They can see the presence or absence of parameters in any equation.

The next step is to create a table of the data in the problem. To extract the data from the problem statement requires several steps. The first and extremely important step is to read the problem as a very short story, i.e. to ignore any numbers given. It is in this step that I encourage the students to create a mental video that they will play over and over again to give a context to the data values supplied by the problem.

As the problem is reread several times, the student fills in the following table with the data.

Kinematic data

The Data Table	
Δx	
v_0	
v	
a	
t	

Note that in physics all numbers consist of three pieces: value, sign and units. Typically the sign and units are the source of great difficulty with novice physics students.

There will be problem statements in which data values are not stated explicitly but are implied by the words. To find them, the video must be replayed to see if there are any unknowns presented in this way. Examples of these hidden data values include "rolls from rest," "drops," "reaches maximum height," and "comes to a stop."

Before proceeding further, the student scans this table for validity (did s/he enter the correct information), then for units consistency. If the units are not consistent, e.g., all in the SI system, conversion must be performed now, before the student starts to think about what the problem is asking and what to do to solve it. One of the keys to success in solving physics problems is to focus on one detail at a time. Errors creep in easily when the student tries to multitask.

For example, we solve the following problem (Serway[1], chapter 2, #35, hereafter denoted by chapter-problem number: 2-35):

> A train 400 m long is moving on a straight track with a speed of 82.4 km/h. The engineer applies the brakes at a crossing, and later the last car passes the crossing with a speed of 16.4 km/h. Assuming constant acceleration, determine how long the train blocked the crossing. Disregard the width of the crossing.

The mental video is that of the train entering a crossing, then slowing down until the video ends with the last car having just passed the crossing. Sketches of both the initial and final positions of the train relative to the crossing should be made. Those positions are the two important frames in the video. The initial data table is

The Data Table	
Δx	
v_0	82.4 km/h
v	16.4 km/h
a	
t	

At this point the mental video needs to be replayed to visualize that the train has moved its length, i.e., 400m. The data table now looks like the following:

The Data Table	
Δx	400 m
v_0	82.4 km/h
v	16.4 km/h
a	
t	

A quick scan of the table shows that the units are inconsistent. Since we expect times on the order of seconds/minutes, not hours, we convert the velocities. As we are looking for the time, we put an arrow next to it to remind us of the current focus. The final data table becomes the following:

The Data Table	
Δx	400 m
v_0	82.4 km/h → 22.9 m/s
v	16.4 km/h → 4.56 m/s
a	
t	⟵

The table tells us what to do next; e.g. find the formula that includes all but acceleration. Referring back to the formula table we find that the appropriate choice is equation (3),

$$\Delta x = \frac{1}{2}(v_0 + v)t$$

From here the problem is straight math; the physics is done. Just plug in the numbers, then solve for time.

More complex kinematics

The above problem involved the use of a single table. In this section we deal with more complex problems. We limit ourselves to two-table problems as the extension to more is straightforward. The first case addresses the problem of a single object having multiple accelerations, either subsequent motion in one dimension or two-dimensional projectile motion. The second type of problem we address is the case of two objects having related single-table motions.

First, consider the following problem:

Serway[1], 2-47

> A ranger in a national park is driving at 35.0 mi/h when a deer jumps into the road 200 ft ahead of the vehicle. After a reaction time of t_R, the ranger applies the brakes to produce an acceleration of $a = -9.00$ ft/s^2. What is the maximum reaction time allowed if she is to avoid hitting the deer?

This problem requires the recognition that it is a two-part problem. Multiple sketches of this time-dependent problem would be appropriate. First, the car moves at constant velocity during the reaction time, the time required for the driver to see the deer, decide to brake, then move her foot to the brake pedal. During the second part, the driver is pushing on the brake pedal with constant force. By watching the mental video, the student can identify the two pieces, the fact that the driver is not interacting with the car until the brake pedal is pressed, at which point the car starts to decelerate.

Sketches of critical frames from the video include the following: the car moving at constant velocity, the initial application of the brakes, and finally, the car stopping just in front of the deer. Note that those of us who have solved physics problems for a long time can easily combine all these actions onto a single sketch but it is much easier for the novice student to see what is happening in the problem if the sketches are separated. (Since friction is not introduced until a later chapter, it is not taken into account at this point in the course i.e., the car does not slow down when the driver moves her foot from accelerator to brake).

The appropriate data tables for this problem begin with the data extracted directly from the problem:

Reacting	
Δx_R	
$v_1 = v_0$	35.0 mi/h
t_R	
$\Delta x = v_0 t$	

Braking	
Δx_2	
v_{02}	
v_2	
a_2	-9.00 ft/s^2
t_2	

Note that the expression for Δx with no acceleration is presented as a reminder; it is not in the table of equations so most students forget about it.

Clearly, more needs to be done before we are ready to solve this problem. The information hidden in words is

$$200 \text{ ft} = \Delta x_2 + \Delta x_R \qquad v_{02} = v_1 \qquad v_2 = 0$$

After the units have been converted, the data table pair becomes

Reacting	
Δx_R	
$v_1 = v_0$	35.0 mi/h → 51.3 ft/s
t_R	
$\Delta x = v_0 t$	

Braking	
Δx_2	
v_{02}	51.3 ft/s
v_2	0
a_2	-9.00 ft/s^2
t_2	

At this point the question is "Now what do I do?" We must establish a plan of action. The problem asks for the reaction time. Consequently, we must find Δx_R since $t_R = \Delta x_R / v_1$ and there is no occurrence of reaction time other than in the left-hand table. This tells us to find Δx_2 in the right-hand table since the two displacements add to 200 feet. We don't care about t_2. The final table pair with formulas entered to remind us what we are doing is

Reacting	
Δx_R	$= 200 - \Delta x_2$ (second)
$v_1 = v_0$	35.0 mi/h → 51.3 ft/s
t_R	⟵ (finally)
$\Delta x = v_0 t$	

Braking	
Δx_2	⟵ (first)
v_{02}	51.3 ft/s
v_2	0
a_2	-9.00 ft/s^2
t_2	

Again, the data tables lead us along a solution path.

Other types of problems that require more than one data table are projectile motion and multi-object problems. Projectile motion problems need data tables in both *x* and *y*. It typically needs to be pointed out that although displacements, velocities and accelerations in *x* and *y* are different, the time must be the same; the object "carries" a clock. An example of a multi-object problem would be the following problem:

Serway[1] 2-50

> A young woman named Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by dragging with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. If Stan moves with a constant acceleration of 3.50m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2, find (a) the time it takes Kathy to overtake Stan, (b) the distance she travels before catching him, and (c) the speeds of both cars at the instant she overtakes him.

Both Kathy and Stan need individual tables. The connection between the two is the relationship between the times of travel as well as the common distance traveled. These relationships are most easily seen by the student by replaying and analyzing the mental video that goes with the problem.

The table is a visual target into which to place numbers in an organized, repeatable fashion and from which to find numbers. Sketches force the student to identify critical frames in the mental video. Hence, points of interest become clear. Tables and multiple sketches can clarify the thinking process.

The Event Model

Since the data table includes the assumption of constant acceleration, multiple data tables are necessary whenever the acceleration changes. This section provides a vocabulary for discussing multi-acceleration systems; e.g., Serway[1], problem 2-47, addressed above. First we borrow from the language of Engineering Thermodynamics.

> "A *property* is any quantity which serves to describe a system. The *state* of a system is its condition as described by giving values to its properties at a particular instant." (Potter[2], page 3)

> "When a system changes from one equilibrium state to another the path of successive states through which the system passes is called a *process*." (Potter[2], page 4)

The properties we consider are the kinematic properties of displacement, time, and velocity. The state at any time includes the values of these properties. If we consider the analogy of a car, the state of the car is read from the following four meters:

i) the odometer presents the magnitude of the displacement,
ii) the speedometer, the magnitude of the velocity,
iii) the clock, the time and
iv) a compass, the direction of the velocity (for 2D problems).

The driver need only read these meters to determine the current state of the system. Acceleration is supplied by an outside force (such as pushing on the brake or accelerator or driving over new terrain) and changes the process that the car will undergo. We term the change point of acceleration an *event*. At this event, the state of the system does not change, but its acceleration does. This change requires the use of a new *transition table*, the tables described above, to describe the evolution of parameters under the new acceleration value and hence the new system process.

There is a corresponding mental video for the event/process model. Under the influence of a single value of acceleration, the system progresses smoothly. We may pause at any point to look at a single frame of this video. The state values in this frame correspond to values in the transition table. One can look at the meter values in the frame to determine the state. An event occurs when they physical environment suddenly changes. In Serway[1], problem 2-47, the start of the problem occurs when the deer is spotted. The acceleration-changing event occurs when the driver's foot presses the brake (assumed to "instantaneously" achieve the proper deceleration). Just before and just after the event, the state of the system does not change; e.g., the time is whatever it is, the ending velocity of the initial constant velocity section is the beginning velocity of the deceleration, and the position is whatever it is the moment the driver's foot hits the brakes.

To further exemplify this concept, consider the following problem:

Serway[1], 3-21

> A car is parked on a cliff overlooking the ocean on an incline that makes an angle of 24.0º below the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s^2 for a distance of 50.0 m to the edge of the cliff. The cliff is 30.0 m above the ocean. Find (a) the car's position relative to the base of the cliff when the car lands in the ocean, and (b) the length of time the car is in the air.

At the start of the problem, the car is stopped, resting on an incline. After a one-dimensional roll down the incline, the event of the car leaving the cliff happens. At this event, the one-dimensional problem becomes two-dimensional; the net force is no longer down the hill, but is now due solely to gravity. A subsequent event occurs when the car lands in the ocean. If asked for the final velocity of the car, I have observed that most students will answer "zero." They missed the second event so we need ask the question a better way, such as "what is the car's velocity just before it hits the water?"

The event concept, described above, provides a structure in which to analyze complex problems by reducing them to simpler connected problems. By identifying events throughout the problem statement, the student can recognize how the problem breaks into its component parts. The visual aid of separate tables for each process helps clarify the task at hand, as do separate sketches at critical points in time.

Dynamics

The application of Newton's laws presents a different visual challenge. Motion is involved, but only in problems involving kinematics as well is there a mention of time. Hence, typically, a single initial sketch is appropriate. Serway[1] presents a clear problem-solving strategy for solving Newton's Law problems. He includes the typical sketch and free-body diagrams in his methodology. One addition I would make is that of arrows pointing in the assumed direction of acceleration in the sketch as well as the free body diagrams. These arrows provide guidance to the students about what is happening in the problem. For instance, in hanging mass problems, students typically set the tension $T=mg$. The acceleration arrow provides a visual reminder that there is acceleration and that it is positive downward. The correct formula is then $mg-T=ma$. Additionally, through creation of one of the coordinate axes in the direction of the acceleration, Newton's Second Law equation has no extra minus signs resulting from an arbitrary choice in positive axis. Once problem solutions start to get complicated, I have found that if we assume that all letters stand for positive quantities, negative answers need to be investigated: Was there a math error? Was there an assumption error? Again, minus signs and units form the bulk of student difficulty in this course.

Application of the methodology seems straightforward until you get to the application of Newton's second law. Whenever the problem is two-dimensional, the students must add components of the forces, a task difficult for most of my students. For this step I rely on the use of visual clues in a table, the force table. I split the construction into two pieces. First, the students should just concentrate on reading the free-body diagram and getting the correct trig entries into the table. Once they have scanned the table for proper entries, then the students can worry about the actual numbers. Early use of calculators masks errors.

Conservation of energy

As another example of the use of tables and multiple diagrams in solving mechanics problems, we address solving problems using the principal of conservation of energy. Like the earlier kinematics problems, these problems involve evolution over time of one or more objects.

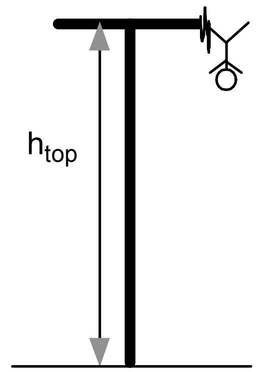
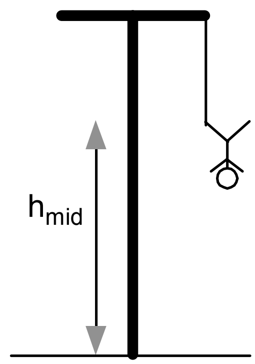
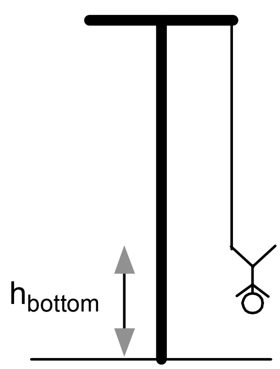
Again, a mental video is appropriate. As the video is replayed in the student's mind, the student should identify "interesting" points in time and/or space. Sketches of these points should be generated with corresponding entries including potential, kinetic and total energies into an energy table, as shown below. The clue to solving these equations is tracking the total energy (the last column in the energy table) throughout the problem. The following question must be answered: "Is energy conserved?" For most beginning problems, it is.

For example, consider the following energy problem (essentially Serway[1], 5-69)

> In the dangerous "sport" of bungee jumping, a daring student jumps from a platform with a specially designed elastic cord attached to his ankle. The unstretched length of the cord is 25.0 m, the student weighs 700 N, and the platform is 36.0 m above the surface of a ground below. Calculate the required force constant of the cored if the student is to stop safely.

We have combined the sketch with the energy data in the table below. The sketch helps identify the meaning of the various parameters in the data table and, therefore, where to put the numbers.

Conservation of Energy Data Tables

Position	KE	PE_g	PE_{spring}	$E_{total} = KE + \Sigma PE$
Beginning of jump 	$\frac{1}{2}mv_{top}^2$	mgh_{top}	0 (cord is slack)	$\frac{1}{2}mv_{top}^2 + mgh_{top}$
Cord is just taut 	$\frac{1}{2}mv_{mid}^2$	mgh_{mid}	0 (cord is in equilibrium position)	$\frac{1}{2}mv_{mid}^2 + mgh_{mid}$
Bottom of jump 	0	mgh_{bottom}	$\frac{1}{2}k(\Delta y)^2$ (Δy is the amount of stretch in the cord)	$mgh_{bottom} +$ $\frac{1}{2}k(\Delta y)^2$

The next step is to enter the numbers from the problem statement into the table. Reviewing the mental video shows that the initial and final velocities, hence kinetic energies, are zero. Since the cord is just flopping around until it first becomes taut, it is not involved in the energy equation until the point above identified as the midpoint. Finally, the initial and final total energies are recognized as being the same because there is no loss of energy in this problem. The student then knows all but the value of *k*.

$$\frac{1}{2}k\left(\Delta y\right)^2 = mg(h_{top} - h_{bottom})$$

The midpoint provides a critical point in the visualization of the problem, but turns out to have no explicit impact on the solution.

Conservation of Momentum

As a last example of the use of tables and multiple diagrams in solving mechanics problems, we address the concept of conservation of momentum. Typical conservation of momentum problems include before and after problems and the collision itself: The objects are prepared for collision, they collide, they continue on their way after collision.

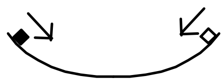
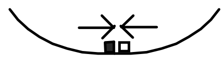
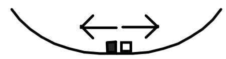
Consider the following problem:

Serway[1] 6-46

> Two blocks of masses m_1=2.00 kg and m_2 = 4.00 kg are each released from rest at a height of 5.00 m on a frictionless track and undergo an elastic head-on collision. (a) Determine the velocity of each block just before collision. (b) Determine the velocity of each block immediately after the collision. (c) Determine the maximum heights to which m_1 and m_2 rise after the collision.

Serway[1] essentially leads the student through the solution in the problem statement by asking the questions in the order given in the problem. First, both blocks obey conservation of energy equations to prepare them for the collision. Then the collision takes place, assumed to be a single event in time and space. Finally, the student must solve two conservation of energy equations to find the heights reached by the blocks. Note how the table with diagrams and equations clarifies the problems that need to be solved.

Collision Data Tables

<table>
<tr><th>Initial configuration</th><th>Just before collision</th><th>Just after collision</th><th>Final configuration</th></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Energy is conserved
$\frac{1}{2}m_1v_{1i}^2 + m_1gh_{1i} =$
$= \frac{1}{2}m_1v_{1f}^2 + m_1gh_{1f}$
and
$\frac{1}{2}m_2v_{2i}^2 + m_2gh_{2i} =$
$= \frac{1}{2}m_2v_{2f}^2 + m_2gh_{2f}$
with
$h_{1f} = 0$, $h_{2f} = 0$,
$v_{1i} = 0$, $v_{2i} = 0$</td><td colspan="2">Energy is conserved since the collision is elastic
$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}v_{1i}^2 = \frac{1}{2}m_1v_{1f}v_{1i}^2 + \frac{1}{2}m_2v_{2f}v_{1i}^2$

Momentum is conserved
$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$</td><td>Energy is conserved for both problems

$\frac{1}{2}m_1v_{1i}^2 = m_1gh_{1f}$
$\frac{1}{2}m_2v_{2i}^2 = m_2gh_{2f}$

with $h_{1i} = h_{2i} = 0$ and v_{1i} and v_{2i} being the post collision initial velocities.</td></tr>
</table>

A visual trap – a caveat

The analysis of diagrams must be made carefully. In conservation of energy problems, the higher the object is on the diagram, the higher the potential energy. The bottom of the problem is typically taken as the position of $y = 0$. However, vertical springs do not behave like gravitational potential energy. Their potential energy is zero at the equilibrium point and positive both above and below equilibrium. Hence, care must be taken to construct and interpret diagrams correctly.

Conclusion

In order to address the learning needs of a diverse group of students, we have used Gardner's theory of multiple intelligences to formulate teaching strategies and provide tools for the students. The use of multiple diagrams and tables, invoking the visual spatial intelligence, appears to improve our students' ability to solve mechanics problems. Each table and diagram forces the student to focus on a particular aspect of a problem instead of trying to solve it all at once. The tables and diagrams clarify paths to solution by identifying each step in a visually identifiable object on the solution page.

The tasks we now set ourselves are, first, to attempt to quantify the results of our use of visual-spatial intelligence in solving physics problems. In other words, we hope to make a quantitative comparison of the grades of physics students who have learned the strategies discussed here with those of physics students who have learned the more traditional strategies. We are only now in possession of sufficient data, after two years of consciously applying the theory of multiple intelligences in the classroom, to make such comparisons possible. Although the formal data analysis has yet to be performed, I have found from tutoring students from other sections of the same course that my students have a firmer grasp on what equation to use to solve a problem. I have gotten much positive feedback from my students on the effectiveness of the table approach. I have received essentially no feedback on the mental video. Since even sketching is often not performed by novices[3], perhaps visualization is too much to expect from my students without explicit classroom training. One of our EET faculty members experienced the same frustration when trying to help his juniors and seniors visualize. This issue clearly needs to be addressed in future research as well as even more attention given in the classroom.

The second task is to explore and perhaps implement other strategies suggested by Gardner's theory, not only in physics, but also in the other classes we teach. We have found that our use of Gardner's theory keeps our teaching fresh; we are constantly revising our lessons and our means of teaching, and the students seem to respond positively.

We also welcome the opportunity to discuss the application of Gardner's theory in the college classroom with like-minded colleagues. We hope that this paper will stimulate such discussion.

ENDNOTES

1. Serway, Raymond A. and Faughn, Jerry S., College Physics, 5^{th} ed., Harcourt College Publishers, 1999.

2. Potter, Merle C. and Somerton, Craig W, Engineering Thermodynamics, Schaum's Outline Series, McGraw Hill, 1993

3. Wankat, Phillip C. and Oreovicz, Frank S., Teaching Engineering, McGraw-Hill, 1993.

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