

Solving Oblique Triangles

Revised

MTH 122

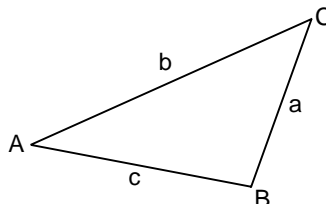
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The table below describes how to “solve” oblique triangles. Solving a triangle means finding all angles and lengths of sides.

The angle A is opposite side a .

The angle B is opposite side b .

The angle C is opposite side c .



You will need the following:

The Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The sum of angles: $A+B+C=180$

The Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Notes:

There can be at most one obtuse angle in a triangle.

The longest side is across from the largest angle.

| Case # | Know | Problem solution |
|--------|--|--|
| 0 | 3 angles | No unique solution – all similar triangles work. |
| 1 | 2 angles, 1 side | First find the third angle using $A+B+C=180$ Then use the Law of Sines twice to find the other two sides.. |
| 2 | 1 angle, 2 sides The angle is <i>across</i> <i>from</i> one of the sides | Use the Law of Sines to find the angle across from the known side not across from the known angle. Assume it is side b . There are three possible results: $\sin(B)>1$ No solution, b is too short $\sin(B)=1$ The triangle is a right triangle. $\sin(B)<1$ There are two solutions to the arcsine, B and $180-B$, so there are two possible triangles. Once two angles are known, find the third from $A+B+C=180$. Use the Law of Sines to find the third side. |
| 3 | 1 angle, 2 sides The angle is <i>between</i> the two sides | Use the Law of Cosines to find the third side, the one opposite the given angle. Since the only possible obtuse angle is across from the longest side, find the largest angle using $A+B+C=180$ <i>after</i> finding the other, smaller unknown angle using the Law of Sines. |
| 4 | 3 sides | Find the angle opposite the largest side using the Law of Cosines. Note that the arccosine returns angles between 0 and 180 so the answer is unique. Then use the Law of Sines followed by $A+B+C=180$ to find the other two angles. Note that the only possible obtuse angle is across from the longest side, so the remaining two unknown angles are acute. |