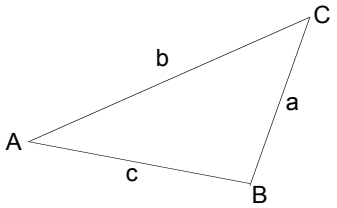
Solving Oblique Triangles
Revised - Spring 2010
MTH 122

The table below describes how to "solve" oblique triangles. Solving a triangle means finding all angles and lengths of sides.

The angle A is opposite side a.
The angle B is opposite side b.
The angle C is opposite side c.



You will need the following:

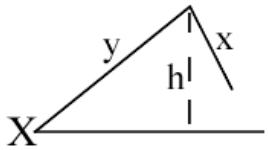
The Law of Sines: $\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{c}$

Or, equivalently: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

The Law of Cosines:
$a^2=b^2+c^2-2bc\cos A$
$b^2=c^2+a^2-2ca\cos B$
$c^2=a^2+b^2-2ab\cos C$

The sum of angles: $A+B+C=180$

Notes:
There can be at most one obtuse angle in a triangle.
The longest side is across from the largest angle.

Case #	Know	Problem solution
0	3 angles	No unique solution – all similar triangles work.
1	2 angles, 1 side	First find the third angle using A+B+C=180 Then use the Law of Sines twice to find the other two sides.
2	1 angle, 2 sides The angle is *across from* one of the sides 	Given x, y, and X, use the Law of Sines to find the angle Y across from side y. Let $h=y\sin(X)$. There are four possible results: x<h No solution, x is too short. x=h The triangle is a right triangle. y>x>h There are two solutions to the arcsine, Y and 180-Y, so there are two possible triangles. Find both. x>y>h There is only one angle Y. 180-Y is negative. Once two angles are known, find the third from A+B+C=180. Use the Law of Sines to find the third side.
3	1 angle, 2 sides The angle is *between* the two sides	Use the Law of Cosines to find the third side, the one opposite the given angle. Since the only possible obtuse angle is across from the longest side, find the largest angle using A+B+C=180 *after* finding the other, smaller unknown angle using the Law of Sines.
4	3 sides	Find the angle opposite the *largest* side using the Law of Cosines. Note that the arccosine returns angles between 0 and 180 so the answer is unique. Then use the Law of Sines followed by $A+B+C=180$ to find the other two angles. Note that the only possible obtuse angle is across from the longest side, so the remaining two angles are acute.