MTH 232 **Exam 3** **December 2009**

Prof. Townsend

Name _____

Instructions:
The exam is open note, open book, work alone, iPod listening is fine. As usual, I want to see details of your work.

Problems:

1) Find the derivative of $y = \sin^2(x^3 - 1)$

$$\frac{dy}{dx} = 2\sin(x^3-1)\frac{d}{dx}\sin(x^3-1)$$
$$\frac{dy}{dx} = 2\sin(x^3-1)\cos(x^3-1)\frac{d}{dx}(x^3-1)$$
$$\frac{dy}{dx} = 2\sin(x^3-1)\cos(x^3-1)(3x^2)$$
$$\frac{dy}{dx} = 6x^2\sin(x^3-1)\cos(x^3-1)$$

2) Find the derivative of $y = \ln\left(\frac{1-3x^2}{x+1}\right)^5$

$$y = \ln\left(\frac{1-3x^2}{x+1}\right)^5 = 5\ln\left(\frac{1-3x^2}{x+1}\right)^5$$
$$\frac{dy}{dx} = 5\frac{d}{dx}\left\{\ln\left(\frac{1-3x^2}{x+1}\right)\right\} \quad \text{(log power rule)}$$
$$\frac{dy}{dx} = 5\frac{d}{dx}\left\{\ln\left(1-3x^2\right) - \ln\left(x+1\right)\right\} \quad \text{(log division rule)}$$

$$\frac{dy}{dx} = 5\left\{\left(\frac{1}{1-3x^2}\right)\frac{d}{dx}\left(1-3x^2\right) - \frac{1}{(x+1)}\frac{d}{dx}(x+1)\right\}$$
$$\frac{dy}{dx} = 5\left\{\left(\frac{1}{1-3x^2}\right)(-6x) - \frac{1}{(x+1)}(1)\right\}$$
$$\frac{dy}{dx} = \frac{-30x}{1-3x^2} - \frac{5}{x+1} = \frac{30x}{3x^2-1} - \frac{5}{x+1}$$

Note: It is the argument of the LN that is raised to the 5th power, not the LN itself.
TI: either F2/comDenom your answer or F2/Expand the TI answer.

3) Find the derivative of $y = 7e^{\sqrt{x^2-2}}$

$$\frac{dy}{dx} = 7\frac{d}{dx}e^{\sqrt{x^2-2}}$$

$$\frac{dy}{dx} = 7e^{\sqrt{x^2-2}}\frac{d}{dx}\sqrt{x^2-2}$$

$$\frac{dy}{dx} = 7e^{\sqrt{x^2-2}}\frac{d}{dx}\left(x^2-2\right)^{\frac{1}{2}}$$

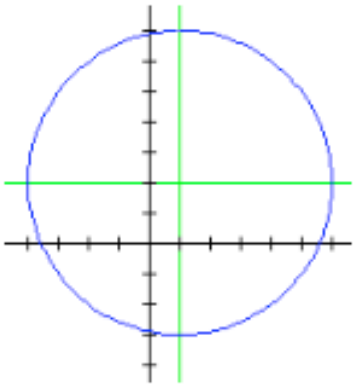
$$\frac{dy}{dx} = 7e^{\sqrt{x^2-2}}\left\{\frac{1}{2}\left(x^2-2\right)^{-\frac{1}{2}}\frac{d}{dx}\left(x^2-2\right)\right\}$$

$$\frac{dy}{dx} = 7e^{\sqrt{x^2-2}}\left\{\frac{1}{2}\left(x^2-2\right)^{-\frac{1}{2}}(2x)\right\} = \frac{7xe^{\sqrt{x^2-2}}}{\sqrt{x^2-2}}$$

4) Determine the center and radius of the following circle. Sketch the circle, indicating the axes and center.

$$(x-1)^2 + (y-2)^2 = 25$$

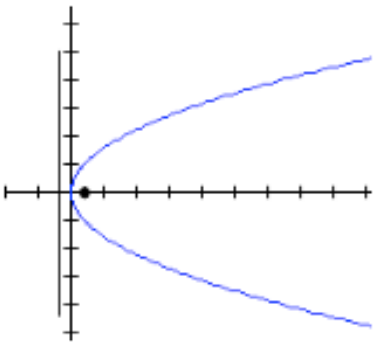
$x^2 + y^2 - 2x - 4y - 20 = 0$ $\quad h = 1$ $\quad k = 2$ $\quad r = \sqrt{25} = 5$



5) Determine the coordinates of the focus and the equation of the directrix of the given parabola. Sketch the parabola, indicating the axes, focus, and directrix.

$$2y^2 - 3x = 0 \qquad y^2 - \frac{3}{2}x = 0$$

$y^2 = \frac{3}{2}x = 4px$ $\quad p = \frac{3}{8}$ $\quad$ focus: $\left(\frac{3}{8}, 0\right)$ $\quad$ directrix: $x = -\frac{3}{8}$

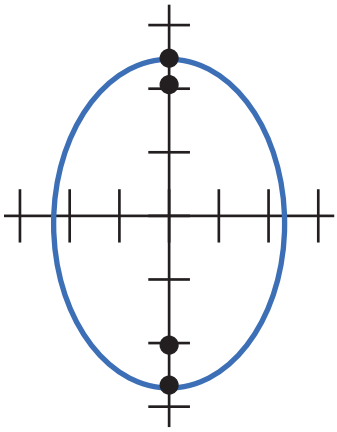


6) Find the equation of the ellipse satisfying the given condition. The center is at the origin. Sketch the ellipse, indicating the axes, vertices, and foci.

Vertex: (0,13), and focus: (0,12)

$a = 13$ $\quad c = 12$ $\quad b = \sqrt{13^2 - 12^2} = 5$

$\dfrac{y^2}{13^2} + \dfrac{x^2}{5^2} = 1$ $\quad 169x^2 + 25y^2 - 4225 = 0$



7) Find the coordinates of the vertices and foci of the given hyperbola. Sketch the hyperbola, indicating the axes.

$y^2 = 9\left(x^2 + 1\right)$ $\quad y^2 - 9x^2 - 9 = 0$ $\quad \dfrac{y^2}{3^2} - x^2 = 1$

$a = 3$ $\quad b = 1$ $\quad c = \sqrt{3^2 + 1^2} = \sqrt{10}$