MTH 352 **Exam 2** **Fall 2009**

Prof. Townsend

Name ____

Extra credit (2 points each) – check your solution using deSolve or substitution for problems number 2, 3, 6, 7, 8. . Note that a deSolve only solution is not acceptable.

Undamped motion

$m = 2 \quad k = 32$

1) (5 points) What is the natural frequency, ω_0, and what does it mean physically?

$$\omega_0 = \sqrt{\frac{k}{m}} = \ldots\ldots$$

2) (15 points) Find the undamped equation of motion, $x(t) = \ldots\ldots$

$$x(t) = C_1 \cos\omega_0 t + C_2 \sin\omega_0 t$$
$$m = 2 \quad k = 32$$
$$x(t) = C_1 \cos 4t + C_2 \sin 4t$$

3) (15 points) Find the position of the mass after two seconds, $x(2)$, given $x(0) = 1$ and $v(0) = 0$.

$$v(t) = -\omega_0 C_1 \sin\omega_0 t + \omega_0 C_2 \cos\omega_0 t$$

$m = 2 \quad k = 32$ $\qquad x(t) = C_1 \cos 4t + C_2 \sin 4t$ $\qquad x(0) = C_1 = 1$

$v(0) = 4C_2 = 0$

$x(t) = \cos 4t$ $\qquad x(2) = \cos 8$

Damped motion

$m = 10 \qquad k = 140 \qquad c = 90$

4) (10 points) What is the discriminant and how does it help us solve damped spring problems?

5) (10 points) What is the value of the damping ratio, ξ, for this problem?

$$\xi = \frac{c}{2m\omega_0} \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

$$m = 10 \qquad k = 140 \qquad c = 90 \qquad \omega_0 = \sqrt{\frac{140}{10}} = \sqrt{14}$$

$$\xi = \frac{c}{2m\omega_0} = \frac{90}{20\sqrt{14}} = 1.2$$

6) (10 points) Determine the type of solution (Case I, II or III). Write down the general form x(t)=… for that case.

$m=10$ $k=140$ $c=90$ $\omega_0=\sqrt{\frac{140}{10}}=\sqrt{14}$ Case I

$\xi=\frac{c}{2m\omega_0}=\frac{90}{20\sqrt{14}}=1.203$ Case I D>0

$x(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}$

7) (15 points) Apply your data to the type of solution found in problem 6.

$x(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}$

solve(10x^2+90x+140=0,x) yields x=-2,-7

$x(t)=C_1e^{-2t}+C_2e^{-7t}$

Forced motion and resonance

$x_1(t)=e^{-2t}$ $x_2(t)=e^{-7t}$

$m=20$ $k=280$ $c=180$

$x_0=0$ $v_0=1$

8) (15 points) The forcing function is $F(t)=10\sin(t)$. The solution is

$$x(t)=\frac{1}{500}\left(110e^{-2t}-101e^{-7t}+13\sin t-9\cos t\right)$$

What is the expression for the particular solution?

$x(t)=C_1x_1(t)+C_2x_2(t)+x_p(t)$

$x_p(t)=\frac{1}{500}(13\sin t-9\cos t)$

What is the differential equation for the problem?

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F(t)$$

$$20\frac{d^2x}{dt^2}+180\frac{dx}{dt}+280x=10\sin(t)$$

What is the value of the Wronskian?

$$W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \dfrac{dx_1(t)}{dt} & \dfrac{dx_2(t)}{dt} \end{vmatrix} = x_1(t)\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}x_2(t)$$

$$W(t) = e^{-2t}\left(-7e^{-7t}\right) - e^{-7t}\left(-2e^{-2t}\right) = -5e^{-9t}$$

9) (5 points) Sinusoidally driven systems can exhibit resonance. What happens to the steady state amplitude of problem 8 if and we modify the applied frequency so that it equals the natural frequency of the damped spring and reduce the damping coefficient to $c = 60/90$?