

Name \_\_\_\_\_

Extra credit (2 points each) – check your solution using deSolve or substitution for problems number 2, 3, 6, 7, 8. Note that a deSolve only solution is not acceptable.

**Undamped motion**

$$m = 2 \quad k = 32$$

- 1) (5 points) What is value of the natural frequency,  $\omega_0$ , and what does it mean physically?
- 2) (15 points) Find the undamped equation of motion,  $x(t) = \dots$
- 3) (15 points) Find the position of the mass after two seconds,  $x(2)$ , given  $x(0) = 1$  and  $v(0) = 0$ .

**Damped motion**

$$m = 10 \quad k = 140 \quad c = 90$$

- 4) (10 points) What is the discriminant and how does it help us solve damped spring problems?
- 5) (10 points) What is the value of the damping ratio,  $\xi$ , for this problem?
- 6) (10 points) Determine the type of solution (Case I, II or III). Write down the general form  $x(t) = \dots$  for that case.
- 7) (15 points) Apply your data to the type of solution found in problem 6.

**Forced motion and resonance**

$$\begin{aligned}x_1(t) &= e^{-2t} & x_2(t) &= e^{-7t} \\m &= 20 & k &= 280 & c &= 180 \\x_0 &= 0 & v_0 &= 1\end{aligned}$$

8) (15 points) The forcing function is  $F(t) = 10 \sin(t)$ . The solution is

$$x(t) = \frac{1}{500} (110e^{-2t} - 101e^{-7t} + 13 \sin t - 9 \cos t)$$

What is the expression for the particular solution?

What is the effective time constant for the transient part of the solution?

What is the differential equation for the problem?

What is the value of the Wronskian?

9) (5 points) Sinusoidally driven systems can exhibit resonance. What happens to the steady state amplitude of problem 8 if and we modify the applied frequency so that it equals the natural frequency of the damped spring?