MTH 352 **Exam 1** **October 22, 2009**

Prof. Townsend

Your Name ____

Your Partner's Name ____

1) (30 points: 6 points each – 3 for linear, 3 for separable) Identify whether the following differential equations are separable, linear, neither or both. To receive full credit you must put the differential equation in the *standard form* for separable and/or linear differential equations, when appropriate. If separable, please identify A(x) and B(x). If linear, please identify p(x) and q(x). If not linear or not separable, tell me how you know.

Do ***not*** solve the equation; just put it in the appropriate form(s).

ID	Exam A	Exam B
a)	$\frac{dy}{dx}-\frac{2x^2}{y}+\frac{y}{x^3}$ nonlinear in y, not separable	$\frac{1}{x}\frac{dy}{dx}+x^2\csc(x)=2xy\cot(x)$ multiply by x $\frac{dy}{dx}+\{-2x^2\cot(x)\}y=\{-x^3\csc(x)\}$ linear, not separable
b)	$\frac{1}{x}\frac{dy}{dx}=y^2(1+x)$ nonlinear, separable $\{x(1+x)\}dx-\left\{\frac{1}{y^2}\right\}dy=0$	$x\frac{dy}{dx}=\frac{y^2}{1+x}$ nonlinear, separable $\left\{\frac{1}{x(1+x)}\right\}dx-\left\{\frac{1}{y^2}\right\}dy$
c)	$x^2\frac{dy}{dx}+3y=\frac{y}{x}$ both $\frac{dy}{dx}+\left\{\frac{3}{x^2}-\frac{1}{x^3}\right\}y=0$ $\left\{\frac{3}{x^2}-\frac{1}{x^3}\right\}dx-\left\{\frac{1}{y}\right\}dy=0$	$\frac{dy}{dx}=\frac{3y}{x^2}+\frac{y^2}{x^3}$ nonlinear not separable
d)	$\frac{dy}{dx}-y\tan(x)-\frac{5y\ln(x)}{x}$ both $\frac{dy}{dx}+\left\{-\tan(x)+\frac{5\ln(x)}{x}\right\}y=0$ $\left\{\frac{1}{y}\right\}dy+\left\{-\tan(x)+\frac{5\ln(x)}{x}\right\}dx=0$	$yx\frac{dy}{dx}-2y=x^2y^2$ divide by y $x\frac{dy}{dx}-2=x^2y$ $\frac{dy}{dx}+\{-x\}y=\left\{\frac{2}{x}\right\}$ linear, not separable
e)	$x^2\frac{dy}{dx}-x=xy$ divide by x^2	$x^2\frac{dy}{dx}=xy+x$

	$\frac{dy}{dx} - \frac{1}{x^2} = \frac{1}{x} y$ $\frac{dy}{dx} + \left\{ -\frac{1}{x} \right\} y = \left\{ \frac{1}{x^2} \right\}$ linear, not separable	$x^2 \frac{dy}{dx} - x = xy$ see left = same problem

Note – you can spot both if q(x) for linear is 0.

MTH 352 **Exam 1** **October 22, 2009**

Prof. Townsend

2 & 3) For problems 2 and 3 (20 points each), choose from the following list of eight differential equations. One equation must be solved using separable techniques and another, by using the formalism for solving linear differential equations. You may solve the same equation by two different methods, if appropriate. You may not solve the same problem using the same method as your partner. Don't forget to use your calculator or internet integrator (integrals.com) to perform integrations. Feel free to use one of the PCs in the room.

<table>
<tr>
<td>

a) $\frac{dy}{dx}+x=x(y+1)$

$\frac{dy}{dx}+x=xy+x$

$\frac{dy}{dx}=xy$ both

Separable

$\frac{dy}{y}=xdx$

$A(x)=x$ $\quad B(y)=-\frac{1}{y}$

$\ln|y|=\frac{x^2}{2}+C$

$y=C'e^{\frac{x^2}{2}}$

Linear

$\frac{dy}{dx}-xy=0$

$p(x)=-x$ $\quad q(x)=0$

$I(x)=e^{-\frac{x^2}{2}}$

$y(x)=e^{\frac{x^2}{2}}\{C\}$

</td>
<td>

e) $\frac{dy}{dx}=\frac{1+x^2}{y^2}$

nonlinear

$y^2dy=\{1+x^2\}dx$

$\frac{y^3}{3}=x+\frac{x^3}{3}+C$

$y(x)=\sqrt[3]{3x+x^3+C'}$

</td>
</tr>
<tr>
<td>

b) $x\frac{dy}{dx}-2x^2y=x^2$ divide by x

$\frac{dy}{dx}-2xy=x$

not separable

$p(x)=-2x$ $\quad q(x)=x$

$y(x)=e^{x^2}\left\{C+\frac{1}{2}\int e^{-x^2}2x\,dx\right\}$

$y(x)=e^{x^2}\left\{C-\frac{1}{2}e^{-x^2}\right\}=Ce^{x^2}-\frac{1}{2}$

</td>
<td>

f) $\frac{dy}{dx}=3x^2e^{-y}$

nonlinear

$e^y dy=3x^2dx$

$e^y=x^3+C$

$y(x)=\ln\{x^3+C\}$

</td>
</tr>
</table>

c) $\frac{dy}{dx}-\frac{3}{x}y=\frac{4}{x}$ not separable $I(x)=e^{-3\ln x}=e^{\ln x^{-3}}=x^{-3}=\frac{1}{x^3}$ $y(x)=x^3\left\{C+\frac{1}{2}\int\frac{1}{x^3}\frac{4}{x}dx\right\}$ $y(x)=x^3\left\{C+2\int x^{-4}\,dx\right\}$ $y(x)=x^3\left\{C+2\frac{x^{-3}}{-3}\right\}$ $y(x)=Cx^3-\frac{2}{3}$	g) $\frac{dy}{dx}=2x\left(y^2+1\right)$ nonlinear $\frac{1}{y^2+1}dy=2xdx$ $\tan^{-1}y=x^2+C$ $y(x)=\tan\left(x^2+C\right)$
d) $\frac{dy}{dx}=2e^{-(x+y)}$ nonlinear $e^y dy=2e^{-x}dx$ $e^y=-2e^{-x}+C$ $y(x)=\ln\left(C-2e^{-x}\right)$	h) $\frac{dy}{dx}=x^3-2xy$ not separable $\frac{dy}{dx}+2xy=x^3$ $y(x)=e^{-x^2}\left\{C+\int e^{x^2}x^3\,dx\right\}$ $y(x)=e^{-x^2}\left\{C+\frac{1}{2}e^{x^2}\left(x^2-1\right)\right\}$ $y(x)=Ce^{-x^2}+\frac{1}{2}\left(x^2-1\right)$

4) (10 points) Evaluate *C* for **one** of your solutions in problems 2 and 3 using the condition that $y(1)=1$. Then write the full solution where *C* has been replaced by its value.

5) (10 points) Solve one of the following problems. Do ***not*** choose that same one as your partner.

Example) You are given the following resisters and capacitors: …………..
In making a simple RC series circuits, which components would you choose to produce the minimum response time? Justify your answer.

6) (10 points) Determine whether or not the given "solution" is a solution of the given differential equation. Be sure to show any intermediate steps. You may perform the algebra steps on your TI but you need to write down what you are doing.

	Solution?	D.E.

Exam A	$y = x^2 + \frac{1}{x}$ yes - take derivative of y then plug into LHS of D.E. You get the RHS	$x^2 \frac{dy}{dx} + 1 = 2x^3$
Exam B	$y = xe^{-x^2}$ yes – see above	$e^{x^2} \frac{dy}{dx} + 2x^2 = 1$