Fourier Series Integrals

Prof. Townsend Spring 2010

Note: $\sin n\pi = 0$ $\cos n\pi = \begin{cases} 1 & n\_even \\ -1 & n\_odd \end{cases}$ $\lim_{a\to 0}\frac{\sin ax}{a} = x$

$n \neq m$ (trig identities are on pages 554-555)

$$\int_{-\pi}^{\pi}\sin nx\sin mxdx = \left\{\frac{\sin(n-m)x}{2(n-m)} - \frac{\sin(n+m)x}{2(n+m)}\right\}\Bigg|_{-\pi}^{\pi} = 0 \qquad \text{E.39}$$

$$\int_{-\pi}^{\pi}\sin nx\cos mxdx = \frac{1}{2}\int_{-\pi}^{\pi}\left\{\sin(n+m)x - \sin(n+m)x\right\}dx \qquad \text{20.9 \& 20.11}$$

$$\int_{-\pi}^{\pi}\sin nx\cos mxdx = \left\{-\frac{\cos(n-m)x}{2(n-m)} - \frac{\cos(n+m)x}{2(n+m)}\right\}\Bigg|_{-\pi}^{\pi} = 0$$

$$\int_{-\pi}^{\pi}\cos nx\cos mxdx = \left\{-\frac{\sin(n-m)x}{2(n-m)} + \frac{\sin(n+m)x}{2(n+m)}\right\}\Bigg|_{-\pi}^{\pi} = 0 \qquad \text{E.41}$$

$n = m$

$$\int_{-\pi}^{\pi}\sin^2 nxdx = \frac{1}{2}\int_{-\pi}^{\pi}(1-\cos 2nx)dx \qquad \text{20.26}$$

$$\frac{1}{2}\int_{-\pi}^{\pi}(1-\cos 2nx)dx = \frac{1}{2}\left\{x - \frac{\sin 2nx}{2n}\right\}\Bigg|_{-\pi}^{\pi} = \pi$$

$$\int_{-\pi}^{\pi}\sin nx\cos nxdx = \frac{1}{n}\int_{-\pi}^{\pi}\sin nx\cos nx(ndx)$$

$$\frac{1}{n}\int_{-\pi}^{\pi}\sin nx(\cos nx(ndx)) = \frac{\sin^2 nx}{2n}\Bigg|_{-\pi}^{\pi} = 0$$

$$\int_{-\pi}^{\pi}\cos^2 nxdx = \frac{1}{2}\int_{-\pi}^{\pi}(1+\cos 2nx)dx \qquad \text{20.26}$$

$$\frac{1}{2}\int_{-\pi}^{\pi}(1+\cos 2nx)dx = \frac{1}{2}\left\{x + \frac{\sin 2nx}{2n}\right\}\Bigg|_{-\pi}^{\pi} = \pi$$