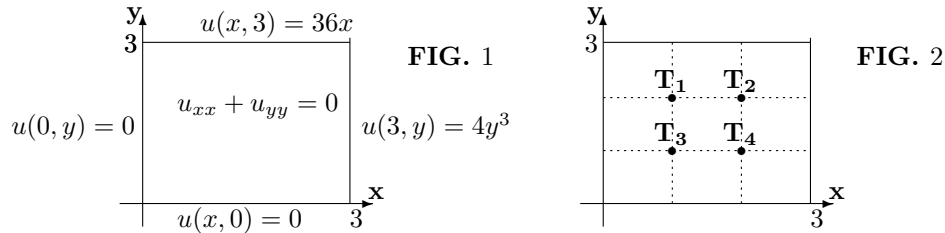


M344 - Lab #3: The Steady-State Heat Equation

Due date: Monday, December 6, 2010

You have been asked to find the steady-state temperature inside a square of size $3m \times 3m$. The boundary temperatures are shown in Figure 1.



Do each of the following:

- Let $\Delta x = \Delta y = 1m$ and use the numerical method described in class (and in Lecture 18) to find approximate values of the temperature $u(x, y)$ at the four points T_1, T_2, T_3 , and T_4 shown in Figure 2. **Show all of your work.**
- Write out carefully the series solution $u(x, y)$ for this problem. It should be the sum of two of the series given in Lecture 18 (you are NOT being asked to derive them). Also write out the integral formulas for the coefficients of both series, using the boundary conditions given in Figure 1.
- The series solution in Problem 2 was programmed in MAPLE, and produced the following values:

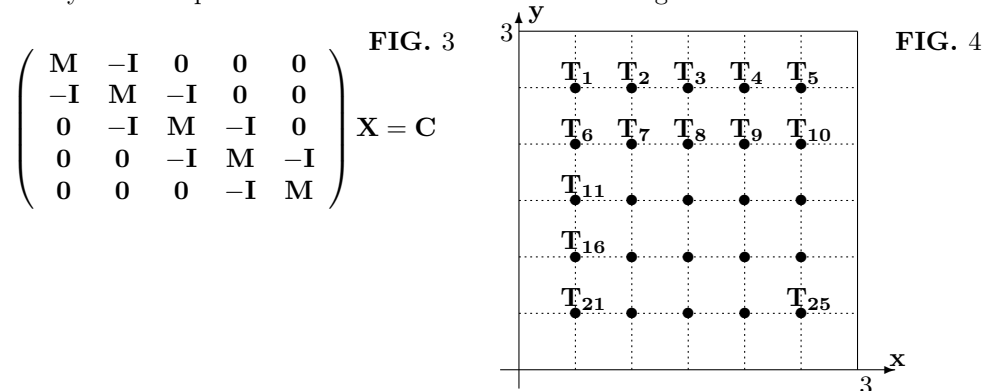
$$u(1, 1) = 8.152, \quad u(2, 1) = 12.018, \quad u(1, 2) = 20.018, \quad u(2, 2) = 34.920.$$

Assuming these values are correct to 3 decimal places, find the percent error in each of your four temperatures T_1, T_2, T_3 and T_4 .

- If Δx and Δy are both reduced to $\frac{1}{2}m$, there will be 25 unknown temperatures T_1, T_2, \dots, T_{25} arranged as shown in Figure 4. If the equations for each T_i are written, using the difference formula

$$4u(x, y) = u(x + \Delta x, y) + u(x - \Delta x, y) + u(x, y + \Delta y) + u(x, y - \Delta y),$$

the system of equations in matrix form will look like Figure 3.



where \mathbf{X} is the column vector $[T_1, T_2, \dots, T_{25}]$ and \mathbf{C} is the column vector $[C_1, C_2, \dots, C_{25}]$ of constant right-hand sides. \mathbf{M} , $-\mathbf{I}$, and $\mathbf{0}$ are each 5×5 matrices. (The 25×25 matrix \mathbf{M} is called a **block matrix** and is easy to set up in MAPLE). Write out equations for the first 5 temperatures T_1, T_2, \dots, T_5 and find the exact form of the three 5×5 matrices \mathbf{M} , $-\mathbf{I}$, and $\mathbf{0}$. **Show all of your work.**

- Write a short introduction to the Lab on the first page inside the cover page.