M242 - DIFFERENTIAL EQUATIONS NOTES **Fall 2008**

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Text: Differential Equations in Science and Engineering,
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Prerequisites: M144 and M145.

Course Summary: This is an introductory course in Differential Equations. It assumes that you have had the first two semesters of Calculus. Since an introduction to differential equations is done at the end of M145, you should have seen much of the material covered in the first two chapters of the text. In particular, the solution of first-order differential equations by the method of separation of variables should be a review. If you did not take Calc II at the University of Hartford, it will be necessary to read Chapters 1 and 2 of the text *very carefully* to make sure you know this material well before the first exam. The general topics to be covered are: first-order differential equations, second-order linear differential equations, systems of first-order d.e.s (including an introduction to matrix methods), phase plane analysis of linear systems, applications to nonlinear systems, and Laplace transforms.

Grading:

Webwork & Homework	15%	Midterm Exams	50%
Labs	15%	Final Exam	20%

Homework: Problems from the text and/or Webwork will be assigned during each class period. You will be given a chance to ask questions about these problems at the beginning of each class. Some homework will be collected and graded, and the Hour Exams will contain questions similar to the homework problems.

Computer Projects (Labs): There will be 2 or 3 projects which involve putting together some of the concepts studied in the course. You will definitely need a TI-89 calculator. The computer software MAPLE can also be used for the projects. It is available in the computer lab in Dana. You will be shown the instructions necessary to do whatever is required. **NO LATE LABS WILL BE ACCEPTED**.

Assessment: Math majors (and others) taking this course will be expected to substantially increase their ability to model real-world problems using mathematics. This skill will be evaluated in the homework and exams, and especially in the Labs.

Date	Sections	Material to be Covered
Sep. 3	1.1, 1.2, 2.1	Introduction to differential equations, separable equations
8	2.2	Slope fields
10	2.3,2.4	Linear d.e.s, Existence & uniqueness
15	2.5	Numerical methods
17	2.6	Autonomous d.e.s, phase line, bifurcation
22	2.7	Applications, review
24		**EXAM 1** (Chap. 1 and 2.1-2.5)
29	3.1	Second-order d.e.s
Oct. 1	3.2	Mass-spring equation
6	3.3	Nonhomogeneous second order d.e.s
8	3.4	Driven mass-spring systems, resonance
13		Applications
15	3.6	Phase planes, review
20		**EXAM 2** (2.6-2.7, 3.1-3.4)
22	5.1,5.2	Introduction to systems, matrix algebra
27	5.3	Eigenvalues and eigenvectors
29	5.4	Analytic solutions of linear systems
Nov. 3		Trace-determinant plane
5	6.1	Phase planes for systems
10	6.2	Linearization of nonlinear systems, review
12		**EXAM 3** (3.6, Chap. 5)
17	4.1	Intro. to Laplace transforms
19		**Thanksgiving Vacation**
24	4.2	Computation of transforms, theorems
26		Examples
Dec. 1	4.3	Unit step and delta function
3	4.4	Examples, convolution
8		Applications
10		Review

FINAL EXAM

Wed., Sept. 3, 2008 Notes

Read Sections 1.1, 1.2, and 2.1 in the text. Webwork Assignment #1 is posted, and will be due 9/13/08.

Definitions

1. A **differential equation** is any equation containing an unknown function and one or more of its derivatives.

2. If the unknown function in a differential equation depends on a single independent variable, the equation is called an **ordinary differential equation** (o.d.e.); otherwise, it is called a **partial differential equation** (p.d.e.).

3. The **order** of a differential equation is the order of the highest derivative appearing in it.

4. A **solution** of an o.d.e. is a function defined on some interval of the independent variable, which when substituted into the equation along with its derivatives, makes the equation an identity in the independent variable.

5. A differential equation is said to be **linear** in the unknown function if the unknown function and its derivatives appear in the equation only to the first power.

6. The most general first order o.d.e. can always be written in the form $x'(t) = F(t, x(t))$.

   The equation is called **separable** if $F(t, x)$ can be factored into a product of the form $g(t) \cdot h(x)$.

   The equation is called **linear** if it can be written in the "standard" form: $x'(t) + p(t)x(t) = q(t)$ for some arbitrary function p and q.

7. A first-order **initial-value problem** (I.V.P.) consists of a first order o.d.e. $x' = F(t, x)$ together with a single initial condition of the form $x(t_0) = x_0$.

8. If $x' = g(t)h(x)$ is a separable equation, and $\bar{x}$ is a constant such that $h(\bar{x}) = 0$, then $\bar{x}$ is called a **constant solution** of the d.e.

Mon., Sept. 8, 2008 Notes

Note: The dates on the Syllabus have been corrected - they were all off by 2.

Read Sections 2.1 and 2.2, Webwork 2 is up.

In Problem 6 on Webwork 1, assume constant acceleration. Find x' and x by integrating x''. Convert everything to feet/second.

Two Applications of Separable o.d.e.s

Newton's Law of Cooling

A small body of temperature $T(t)$ in a room, with constant ambient temperature A, cools at a rate proportional to the temperature difference: $T'(t) = k(T(t) - A)$.

Logistic population growth

The rate of growth of a population $P(t)$ is given by $P'(t) = rP(t)(1 - P(t)/N)$, where r is called the *intrinsic growth rate* and N is called the *carrying capacity.*

Chap. 2.2: Slope fields for $x' = f(t, x)$

At each point (t_i, x_i) of a grid of points in the (t, x)-plane a short line segment with slope $f(t_i, x_i)$ is drawn, showing how solutions of the differential equation move through the plane.

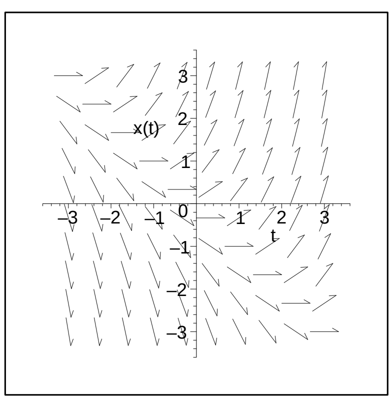
MAPLE can be used to draw a slope field for a first order d.e. $x' = f(t, x)$, by using the instructions

```
with(DEtools):
DEplot({o.d.e.},[unknown function],range of ind.variable,options);
```

As an example, if the `DEplot` instruction

```
DEplot({diff(x(t),t)=x(t)+t},[x(t)],t=-3..3,x=-3..3,dirgrid=[10,10],color=BLUE);
```

is executed, it produces the slope field shown below.



The option `dirgrid=[M,N]` allows you to say how many grid points to use in the horizontal and vertical directions, respectively.

See your text for the steps needed to draw a slope field using the TI89. You may want to use this on the exams.