Name __________________________________________________________

Instructions:
The exam is open note, open book, work alone, iPod listening is fine. As usual, I want to see
details of your work.

Problems:

1) Find the derivative of \( y = \sin^2(x^3 - 1) \)

\[
\frac{dy}{dx} = 2 \sin(x^3 - 1) \frac{d}{dx} \sin(x^3 - 1)
\]

\[
\frac{dy}{dx} = 2 \sin(x^3 - 1) \cos(x^3 - 1) \frac{d}{dx}(x^3 - 1)
\]

\[
\frac{dy}{dx} = 2 \sin(x^3 - 1) \cos(x^3 - 1)(3x^2)
\]

\[
\frac{dy}{dx} = 6x^2 \sin(x^3 - 1) \cos(x^3 - 1)
\]

2) Find the derivative of \( y = \ln \left( \frac{1-3x^2}{x+1} \right)^5 \)

\[
y = 5 \ln \left( \frac{1-3x^2}{x+1} \right)
\]

\[
\frac{dy}{dx} = 5 \frac{d}{dx} \ln \left( \frac{1-3x^2}{x+1} \right) \\
\text{(log power rule)}
\]

\[
\frac{dy}{dx} = 5 \frac{d}{dx} \left\{ \ln(1-3x^2) - \ln(x+1) \right\} \\
\text{(log division rule)}
\]

\[
\frac{dy}{dx} = 5 \left\{ \frac{1}{1-3x^2} \frac{d}{dx}(1-3x^2) - \frac{1}{x+1} \frac{d}{dx}(x+1) \right\}
\]

\[
\frac{dy}{dx} = 5 \left\{ \frac{1}{1-3x^2} (6x) - \frac{1}{(x+1)} (1) \right\}
\]

\[
\frac{dy}{dx} = \frac{-30x \cdot 5}{3x^2 - 1} = \frac{30x}{3x^2 - 1} - \frac{5}{x+1}
\]

Note: It is the argument of the LN that is raised to the 5th power, not the LN itself.

TI: either F2/comDenom your answer or F2/Expand the TI answer.
3) Find the derivative of \( y = 7e^{\sqrt{x^2-2}} \)

\[
\frac{dy}{dx} = 7 \frac{d}{dx} e^{\sqrt{x^2-2}} \\
\frac{dy}{dx} = 7e^{\sqrt{x^2-2}} \frac{d}{dx} \sqrt{x^2-2} \\
\frac{dy}{dx} = 7e^{\sqrt{x^2-2}} \frac{d}{dx} (x^2 - 2)^{\frac{1}{2}} \\
\frac{dy}{dx} = 7e^{\sqrt{x^2-2}} \left\{ \frac{1}{2} (x^2 - 2)^{-\frac{1}{2}} \frac{d}{dx} (x^2 - 2) \right\} \\
\frac{dy}{dx} = 7e^{\sqrt{x^2-2}} \left\{ \frac{1}{2} (x^2 - 2)^{-\frac{1}{2}} (2x) \right\} = \frac{7xe^{\sqrt{x^2-2}}}{\sqrt{x^2-2}}
\]

4) Determine the center and radius of the following circle. Sketch the circle, indicating the axes and center.

\[(x - 1)^2 + (y - 2)^2 = 25 \]

\( x^2 + y^2 - 2x - 4y - 20 = 0 \)

\( h = 1 \quad k = 2 \quad r = \sqrt{25} = 5 \)

![Circle Diagram](image)

5) Determine the coordinates of the focus and the equation of the directrix of the given parabola. Sketch the parabola, indicating the axes, focus, and directrix.

\( 2y^2 - 3x = 0 \)

\( y^2 - \frac{3}{2}x = 0 \)

\( y^2 = \frac{3}{2}x = 4px \quad p = \frac{3}{8} \quad \text{focus:} \left( \frac{3}{8}, 0 \right) \quad \text{directrix:} x = -\frac{3}{8} \)

![Parabola Diagram](image)
6) Find the equation of the ellipse satisfying the given condition. The center is at the origin. Sketch the ellipse, indicating the axes, vertices, and foci.
   Vertex: (0,13), and focus: (0,12)

\[
\frac{y^2}{13^2} + \frac{x^2}{5^2} = 1 \quad \quad 169x^2 + 25y^2 - 4225 = 0
\]

7) Find the coordinates of the vertices and foci of the given hyperbola. Sketch the hyperbola, indicating the axes.

\[
y^2 = 9(x^2 + 1) \quad y^2 - 9x^2 - 9 = 0 \quad \frac{y^2}{3^2} - x^2 = 1
\]

\[
a = 3 \quad b = 1 \quad c = \sqrt{3^2 + 1^2} = \sqrt{10}
\]