Quadratics and Parabolas
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MTH 112
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We now look at three variations of the same equation.

\[ y = x^2 - 2x - 3 \]

Define \( a = 1 \)

Define \( b = 2 \)

Define \( c = 3 \)

\[ b^2 - 4ac = 16 \]

\[ \text{solve} \left( ax^2 + bx + c = 0, x \right) \]

\( x = -1 \) or \( x = 3 \)

\[ y = x^2 - 2x + 1 \]

Define \( a = 1 \)

Define \( b = 2 \)

Define \( c = 1 \)

\[ b^2 - 4ac = 0 \]

\[ \text{solve} \left( ax^2 + bx + c = 0, x \right) \]

\( x = 1 \)

\[ y = x^2 - 2x + 5 \]

Define \( a = 1 \)

Define \( b = 2 \)

Define \( c = 5 \)

\[ b^2 - 4ac = -16 \]

\[ \text{solve} \left( ax^2 + bx + c = 0, x \right) \]

false
Now look at where the graphs cross the x-axis.

These points are called by several names;
- the roots of the equations
- the zeros of the equations
- the x-intercepts of the equations.

### Interesting points on the graph:

<table>
<thead>
<tr>
<th>Name</th>
<th>Where is it on the graph?</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-intercept</td>
<td>Where the graph crosses the x axis</td>
<td>y = 0</td>
</tr>
<tr>
<td>y-intercept</td>
<td>Where the graph crosses the y axis</td>
<td>x = 0</td>
</tr>
</tbody>
</table>
| Vertex   | Point on the graph is a Maximum (when $a < 0$) or Minimum (when $a > 0$) | $x_v = \frac{-b}{2a}$
          |                     | $y_v = f(x_v)$ i.e. plug $x_v$ into the $y$ equation |

### The Discriminant: $b^2 - 4ac$
i.e. what’s under the square root symbol

<table>
<thead>
<tr>
<th>Equation</th>
<th>Number of solutions</th>
<th>Graphical realization</th>
</tr>
</thead>
</table>
| $b^2 - 4ac > 0$ | Two solutions of the quadratic formula | Vertex below the x axis $a > 0$
|            |                                      | Vertex above the x axis $a < 0$                      |
| $b^2 - 4ac = 0$ | One solution of the quadratic formula | Vertex on the x axis                                  |
| $b^2 - 4ac < 0$ | No solution of the quadratic formula | Vertex above the x axis $a > 0$
|            |                                      | Vertex below the x axis $a < 0$                      |