This exam covers the material in exams 1 and 2. Do not use Laplace transforms on this exam. You may use deSolve to check your work but not for the problem solution itself. You may use your calculator for all algebra, integrals, and derivatives.

1) (25 points) Solve one the following differential equations with \( y(0) = -4 \) using linear techniques.
   \[
   \frac{dy}{dx} - xy = -3x \quad \frac{dy}{dx} - 3y = 2
   \]

2) (25 points) Solve one the following differential equations with \( y(0) = -3 \) using separation of variables.
   \[
   \frac{dy}{dx} = \frac{x^2 + 4}{y} \quad \frac{dy}{dx} = y^2 x^4
   \]

3) (25 points) Solve one the following differential equations with \( y(0) = 1 \) and \( y'(0) = 0 \).
   \[
   \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 6y = 0 \quad \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0
   \]

4) (25 points) Solve one the following differential equations with \( y(0) = 1 \) and \( y'(0) = 0 \).
   \[
   \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{-4x} \quad \text{where} \quad y_1(x) = e^{-x} \quad \text{and} \quad y_2(x) = e^{2x}.
   \]
   \[
   \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 8y = 7 \quad \text{where} \quad y_1(x) = e^{-2x} \cos(2x) \quad \text{and} \quad y_1(x) = e^{-2x} \sin(2x).
   \]

Random notes:

i) You can define \( y_1(x) \) in the graph menu then use it on the home screen. You can also use F4/1 Define \( y_1(x)=… \) from the home screen. For instance, you might be interested in computing \( y_1(x) \frac{dy}{dx} y_2(x) \). Define both then calculate \( y_1(x) \cdot d(y_2(x),x) \).

ii) \( \cos^2(\theta) + \sin^2(\theta) = 1 \)

iii) If solve() says false, use csolve() to get complex roots.

iv) D<0 problems are much more work than the other two types.