1) The ellipse is defined by the sum of the two lengths, $L_1$ and $L_2$, is a constant which we will call $2a$.

\[ L_1 + L_2 = 2a \]

What is $a$?

So $2a$ is the length of the major axis. Thus $a$ is the length of the semi-major axis.

The two foci are at $(-c,0)$ and $(c,0)$. What about the points at the top and bottom of the y axis? Below is a naming diagram.
Now find out how b fits in.

Draw lines from the foci to the semi-minor axis, b. Since the sum is 2a and the lines are the same length, each line is of length a.

By the Pythagorean theorem, \( a^2 = b^2 + c^2 \). Note that

1) \( |x| \leq a \) and \( |y| \leq b \)
2) \( a > b \)

Vertical Ellipses – just interchange x and y. The rest remains the same.

### Summary

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
</tr>
</thead>
</table>
| Horizontal Ellipse | \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)  
Note: \( |x| \leq a, |y| \leq b \)  
\( a > b \)  
\( a^2 = b^2 + c^2 \)  
Vertices: \((\pm a,0)\)  
Foci: \((\pm c,0)\) | \( \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \)  
Note: \( |x| \leq b, |y| \leq a \)  
\( a > b \)  
\( a^2 = b^2 + c^2 \)  
Vertices: \((0,\pm a)\)  
Foci: \((0,\pm c)\) |

Sample Problems from page 582.

Numbers 3 to 16 – force into the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) or \( \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \). The larger number is a since \( a > b \).

The variable of a tells you which orientation the ellipse has.
<table>
<thead>
<tr>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 )</td>
<td>Major axis = 2a</td>
</tr>
<tr>
<td>Vertices: ((\pm a,0)) = Semi-major axis</td>
<td>Vertices: ((0,\pm a)) = Semi-major axis</td>
</tr>
<tr>
<td>Foci: ((\pm c,0)) Semi-minor axis: ((0,\pm b))</td>
<td>Foci: ((0,\pm c)) Semi-minor axis: ((\pm b,0))</td>
</tr>
</tbody>
</table>

**Problems page 582**

**Problem 4.** \( a = 10 \), \( b = 8 \), \( c = \sqrt{10^2 - 8^2} = 6 \) Horizontal (the \( 10^2 \) is under the \( x^2 \))

**Problem 6.** \( a = 9 \), \( b = 7 \), \( c = \sqrt{9^2 - 7^2} = \sqrt{32} \) Vertical (the \( 9^2 \) is under the \( y^2 \))

**Problem 9.** \( 4x^2 + 9y^2 = 324 \)

There are two problems

1) the right hand side is not 1.
2) the coefficients of \( x^2 \) and \( y^2 \) are not 1.

First divide by 324 to make the right side equal to 1.

Now adjust the coefficients.

\[
\frac{x^2}{324} + \frac{y^2}{9} = 1
\]

Do the division

\[
\frac{x^2}{81} + \frac{y^2}{36} = 1
\]

\( a = 9 \), \( b = 6 \), \( c = \sqrt{9^2 - 6^2} = \sqrt{45} \) Horizontal (the \( 9^2 \) is under the \( x^2 \))

**Problem 17.** \( a = 15 \), \( c = 9 \)

\( a^2 = b^2 + c^2 \) so \( 15^2 = b^2 + 9^2 \)

\( b^2 = 144 \) \( b = 12 \)

\[
\frac{x^2}{225} + \frac{y^2}{144} = 1
\]

Horizontal (the \( 15^2 \) is under the \( x^2 \))

**Problem 18.** \( a = 5 \), \( b = 4 \) \( c = \sqrt{5^2 - 4^2} = 3 \) Vertical since the vertex is on the \( y \) axis.

\[
\frac{x^2}{16} + \frac{y^2}{25} = 1
\]

**Problem 19.** \( a = 17 \), \( c = 8 \) Vertical since the focus is on the \( y \) axis.

\( a^2 = b^2 + c^2 \) so \( 17^2 = b^2 + 8^2 \)

\( b^2 = 225 \) \( b = 15 \)

\[
\frac{x^2}{25} + \frac{y^2}{289} = 1
\]
Problem 20.  \( a = 13, \ c = 5 \quad b = \sqrt{13^2 - 5^2} = 12 \)  Vertical since the focus and vertex are on the y axis.
\[
\frac{x^2}{144} + \frac{y^2}{169} = 1
\]

Problem 21.  \( b = 12, \ c = 8 \quad a = \sqrt{12^2 + 8^2} = \sqrt{208} \)  Horizontal since the focus is on the x axis and the end of the minor axis is on the y axis.
\[
\frac{x^2}{208} + \frac{y^2}{144} = 1
\]

Problem 23.  \( a = 8 \quad (x,y) = (2,3) \)  Horizontal since the vertex is on the x axis.
\[
\frac{2^2}{64} + \frac{3^2}{b^2} = 1
\]
\[
b^2 = \frac{144}{15} = 9.6 \quad b = 3.098
\]
\[
\frac{x^2}{64} + \frac{y^2}{9.6} = 1
\]

Problem 24.  \( c = 2 \quad (x,y) = (-1,\sqrt{3}) \)  Vertical since the focus is on the y axis.
\[
\frac{(-1)^2}{b^2} + \frac{\sqrt{3}^2}{b^2 + 4} = 1 \quad \text{Note:} \quad a^2 = 7.464
\]
\[
b^2 = 2 \quad a^2 = b^2 + 4 = 6
\]
\[
\frac{x^2}{2} + \frac{y^2}{6} = 1
\]

Problem 25.  \( (x,y) = (2,2) \)  and  \( (x,y) = (1,4) \)  
Solve
\[
\frac{2^2}{u} + \frac{2^2}{v} = 1 \quad \text{and} \quad \frac{1^2}{u} + \frac{4^2}{v} = 1
\]
for \( u \) and \( v \)
\[
u > u \]
\[
u > u \]
\[
u = 20 = a^2 \quad \text{Vertical since} \quad v > u
\]
\[
\frac{x^2}{5} + \frac{y^2}{20} = 1
\]