Reread all problems when you think you are done to make sure you answered the question!

Section 1 Tangents and Normals

The slope of the tangent line is $m = \frac{dy}{dx}$. The slope of the normal (perpendicular) line is $m_\perp = -\frac{1}{m}$.

Note that to get to the normal slope you need the tangent line slope.

A) Sometimes you are given the $(x, y)$ point and you need to find the equation of the tangent or normal line.

1) Find the slope of the line.
2) Use the point and the slope to find the equation $y = m \cdot x + b$.
3) Find $b$ then write the equation of the line $y = m \cdot x + b$.
4) Or use the point-slope form $(y - y_{\text{given}}) = m \cdot (x - x_{\text{given}})$.
5) Solve for $y$ then write the equation of the line $y = m \cdot x + b$.

B) Sometimes you are given the slope of the line, $m$. If given $m_\perp$, you need to solve for $m$ from $m = -\frac{1}{m_\perp}$.

1) Take the derivative $\frac{dy}{dx}$.
2) Set it equal to $m$. $m = \frac{dy}{dx}$.
3) Solve for $x$.
4) Plug $x$ into the original equation to find $y$.
5) Find the equation of the requested line as done in A.

Section III Curvilinear Motion

The position is given by the vector $\vec{s} = [x, y]$.

The velocity is given by the vector $\vec{v} = \frac{d\vec{s}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j}$.

The acceleration is given by the vector $\vec{a} = \frac{d\vec{v}}{dt} = \left[ \frac{dv_x}{dt}, \frac{dv_y}{dt} \right] = \left[ \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right]$.

There are three types of problem.

A) You are given $\vec{s}(t) = [x(t), y(t)]$ or $\vec{v}(t) = [v_x(t), v_y(t)]$. Find velocity and/or acceleration. Use the above formulas to get your answer.

B) You are given $y(x)$ and $v_x(t)$. Find velocity and/or acceleration. You will need the chain rule to get $v_y(t) = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{dy}{dx} \cdot v_x(t)$.

Section IV Related Rates

You are given some equation connecting two variables, $y = f(w)$, where $y$ and $w$ are the two variables. Use the chain rule to find the answer. Example:

$y = 3w^2 + w + 9$

$\frac{dy}{dt} = 3(2w \frac{dw}{dt}) + \frac{dw}{dt} = (6w + 1)\frac{dw}{dt}$

You will be given one of the derivatives and a value for $w$. Find the value for the other derivative.
Section V Curve Sketching

You will use the information in this section for the max-min problems. Maxima and minima might occur when the slope is zero. The second derivative tells you which type of situation it is. Check validity by graphing y(x).

<table>
<thead>
<tr>
<th>First Derivative</th>
<th>Second Derivative</th>
<th>Type of situation</th>
<th>Graph example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dx} = 0 )</td>
<td>( \frac{d^2y}{dx^2} &gt; 0 )</td>
<td>Minimum Concave up</td>
<td>![Graph of Minimum Concave Up]</td>
</tr>
<tr>
<td>( \frac{dy}{dx} = 0 )</td>
<td>( \frac{d^2y}{dx^2} &lt; 0 )</td>
<td>Maximum Concave down</td>
<td>![Graph of Maximum Concave Down]</td>
</tr>
<tr>
<td>( \frac{dy}{dx} = 0 )</td>
<td>( \frac{d^2y}{dx^2} = 0 )</td>
<td>Inflection point Concave up on one side, Concave down on the other.</td>
<td>![Graph of Inflection Point]</td>
</tr>
</tbody>
</table>

Section VII Applied Max/Min Problems

There are two types in this section: one variable, \( y(x) \), and two variables, \( y(x,w) \)

A) one equation in one variable \( y(x) \):

1) find \( \frac{dy}{dx} \)

2) Set it equal to zero to find all possible maxima and minima and inflection points.
   \[ \frac{dy}{dx} = 0 \quad \text{determine } x \]

3) Find \( \frac{d^2y}{dx^2} \) then use above chart to determine type of solution.

4) Given \( x \), find \( y \).

B) Two equations in two variables \( f(x,y) \) and \( g(x,y) \):

One equation will have a given number relating the two variables.

Example:

The area of a region is 25. Then \( A = x \cdot y = 25 \).

The other will be the perimeter: \( P = 2x + 2y \). Minimize \( P \).

1) Use the equation with the given number to solve for one variable in terms of the other:
   \[ y = \frac{25}{x} \]

2) Plug it into the other equation to generate a single variable equation:
   \[ P = 2x + 2 \left( \frac{25}{x} \right) \]

3) Use the one variable solution method, A, to finish the problem.
   \[ \frac{dP}{dx} = 2 - \frac{50}{x^2} ; \quad 0 = 2 - \frac{50}{x^2} \rightarrow x = 5 ; \]
   \[ \frac{d^2P}{dx^2} = \frac{100}{x^3} > 0 \quad \text{Minimum} \]
   \[ y = \frac{25}{x} = \frac{25}{5} = 5 ; \quad P = 2x + 2y = 20 \]